

# ROTOR-BEARING STABILITY

by

Edgar J. Gunter, Jr.

Department of Mechanical Engineering  
University of Virginia



*Edgar J. Gunter Jr. is an associate professor of Mechanical Engineering at the University of Virginia. He was with Clark Brothers Company, Olean, New York during the period from 1956 through 1959, and was with Franklin Institute, Philadelphia, Pennsylvania during the period from 1962 through 1965.*

*He received a B.S. (Mechanical Engineering) Degree in 1956 from Duke University, received a M.S. (Engineering Mechanics) Degree in 1961, and a Ph.D. (Engineering Mechanics) Degree in 1965 from University of Pennsylvania.*

*He is a member of several professional societies. He has several publications, technical notes, and technical reports in the area of bearings.*

## ABSTRACT

This paper presents a survey of the various mechanisms that can cause instability in rotor bearing systems. The occurrence of self-excited instability is of particular importance to the manufacturers and users of modern turbomachinery particularly with the present trends towards high speed and loading conditions.

The paper presents stability data on plain and multilobed journal bearings and shows the effect of unbalance and external loading on the nonlinear rotor whirl orbits.

In addition to the stability characteristics of hydrodynamic bearings, instability due to internal friction, aerodynamic cross coupling and seals is also discussed. The paper discusses how many of these instabilities may be avoided by proper bearing and support design.

In addition to the theoretical prediction of rotor whirl orbits, actual case histories of rotor instability are presented.

## INTRODUCTION

The object of this paper is to review some of the mechanisms that can create nonsynchronous whirl motion in a rotor-bearing system. This motion may be manifested by a sub-synchronous or super harmonic motion or combination of various motions and its direction may be forward or backward precession. There are many mechanisms which can cause whirling or non-synchronous precession in a rotor system but the most serious ones of consequence in turbomachinery is the occurrence of self-excited whirling such as caused by hydrodynamic bearing instability, internal friction, or

aerodynamic cross coupling. When these conditions are encountered the vibration may be so severe as to cause destruction of the rotor. It is therefore imperative for the plant engineer to be able to recognize the occurrence of potentially dangerous whirl motion so as to change the rotor operating conditions or speed. It is preferable that whirl motion be taken into consideration in turbomachinery design so as to minimize the possible occurrence of self-excited whirl motion in operation.

As the operating requirements of speed and power becomes increasingly more demanding on turbo machinery, conditions are encountered in which the turbines and compressors are running at many times the rotor first critical speed. When this occurs the system may be susceptible to whirl of many forms. In the past decade whirl motion has become of increasingly more concern to the engineer and considerable literature has been published on various aspects of it. Now that electronic instrumentation has developed to measure rotor motion and analyze the frequency components it is possible to accurately monitor the whirl motion experienced in rotating machinery under actual field conditions.

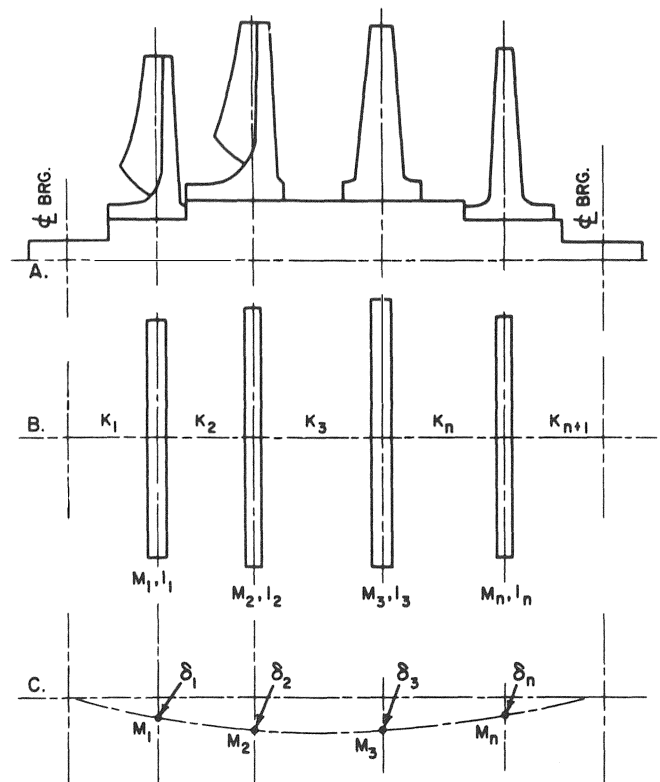


Figure 1. Schematic Diagram of Flexible Rotor.

It is impossible in this presentation to adequately examine all the various aspects of whirling in high speed rotating machinery. It is the purpose of this paper, however to educate and acquaint the engineer with some of the various modes of whirl motion experienced with turbomachinery.

### Description of the System

The usual high-speed rotor may be considered as a continuous elastic body with variable mass and inertia properties along its length as shown in Figure 1A. The shaft usually has attached to it such components as turbine wheels, impeller disks, spacers, etc. In the calculation of rotor critical speeds, stability, and unbalance response, most computer codes treat the rotor as a lumped mass system in which the rotor is represented by an elastic shaft to which is attached  $N$ - mass stations. If the moments of inertia of each section are ignored, then the stations may be considered as concentrated masses as shown in Figure 1c. This assumption leads to the neglect of the gyroscopic moments acting on the rotor which is unacceptable for certain high speed rotors or overhung configurations.

Figure 2 represents a typical cross section of an idealized rotor taken at the  $N$ th station. The total angu-

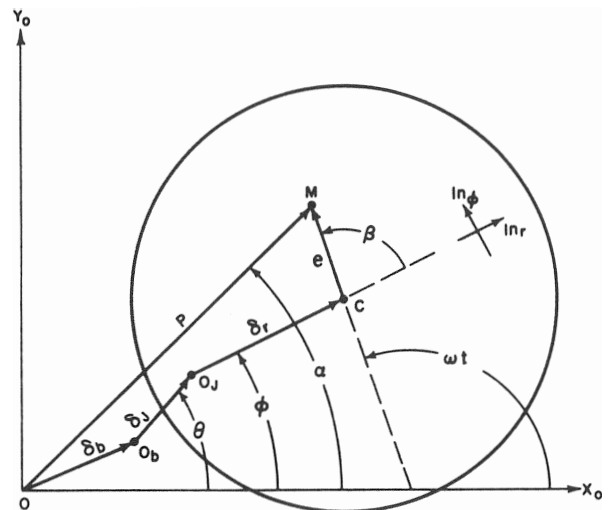


Figure 2. Rotor Cross Section.

lar velocity of the system is given by the time rate of change of a line fixed in the disk and is represented by  $\omega$ . By examination of the configuration of Figure 2,

TABLE 1. FORCES ACTING ON ROTOR-BEARING SYSTEMS (Ref. Rieger)

| Source of Force   | Description  | Application  |
|---|--|--|
| 1. Forces transmitted to foundations, casing, or bearing pedestals. | Constant, unidirectional force<br>Constant force, rotational<br>Variable, unidirectional<br>Impulsive forces<br>Random forces  | Constant linear acceleration.<br>Rotation in gravitational or magnetic field.<br>Impressed cyclic ground—or foundation-motion.<br>Air blast, explosion or earthquake. Nearby unbalanced machinery. Blows, impact.  |
| 2. Forces generated by rotor motion.                                | Rotating unbalance:<br>Residual, or bent shaft.<br>Coriolis forces<br><br>Elastic hysteresis of rotor<br><br>Coulomb friction<br><br>Fluid friction<br><br>Hydrodynamic forces, static.<br><br>Hydrodynamic forces, dynamic.<br>Dissimilar elastic beam<br>Stiffness reaction forces<br><br>Gyroscopic moments | Present in all rotating machinery.<br><br>Motion around curve of varying radius. Space applications. Rotary-coordinated analyses.<br>Property of rotor material which appears when rotor is cyclically deformed in bending, torsionally or axially.<br>Construction damping arising from relative motion between shrunk fitted assemblies.<br>Dry-friction bearing whirl.<br>Viscous shear of bearings.<br>Fluid entrainment in turbomachinery. Windage.<br>Bearing load capacity.<br>Volute pressure forces.<br>Bearing stiffness and damping properties.<br>Rotors with differing rotor lateral stiffnesses.<br>Slotted rotors, electrical machinery, Keyway.<br>Abrupt speed change conditions<br>Significant in high-speed flexible rotors with disks. |
| 3. Applied to rotor   | Drive torque<br>Cyclic forces<br><br>Oscillating torques<br><br>Transient torques<br>Heavy applied rotor force<br><br>Gravity<br>Magnetic field, stationary or rotating.<br>Axial forces   | Accelerating or constant-speed operation<br>Internal combustion engine torque and force components.<br>Misaligned couplings. Propellers. Fans.<br>Internal combustion engine drive.<br>Gears with indexing or positioning errors<br>Drive gear forces<br>Misaligned 3-or-more rotor-bearing assembly.<br>Non-vertical machines. Non-spatial applications.<br>Rotating electrical machinery<br><br>Turbomachine balance piston. Cyclic forces from propeller, or fan. Self-excited bearing forces.<br>Pneumatic hammer.   |

the following definitions of whirl ratio may be stated:

$$\phi/\omega = \text{shaft whirl} = 1 - \text{Synchronous precession or "whirl ratio"} \neq 1 - \text{Nonsynchronous Precession}$$

$$\Theta/\omega = \text{journal whirl ratio}$$

$$\alpha/\omega = \text{system whirl or precession ratio}$$

For the case of steady-state unbalance synchronous precession with isentropic bearings (17), the configuration formed by  $O$ ,  $O_b$ ,  $O_j$ ,  $C$ ,  $N$ , is constant and precesses with an angular velocity equal to the rotor angular velocity  $\omega$ .

Whirling in turbomachinery can be generated by many influences acting on rotor. Table 1 represents a summary of the various forces acting on rotor-bearing systems as compiled by Rieger (48).

#### *Causes of Rotor Whirling in Turbomachinery*

There are many factors which can lead to rotor whirling in turbomachinery. Some of the mechanisms that can induce rotor whirl motion are given as follows:

1. Hysteretic or Internal Friction Damping: a. Shrink fits; b. Friction in gear type couplings; c. Shaft internal damping.
2. Hydrodynamic fluid film bearings and seals.
3. Aerodynamic cross coupling forces.
4. Dry friction whirl (rotor rub).
5. Pulsating Axial Loads and Torques.
6. Asymmetric shafting properties.
7. Entrained fluid in rotor (partially filled centrifuge.)
8. Gyroscopic induced whirling.
9. Subharmonic and superharmonic whirling induced by system nonlinearities.
10. Electromagnetic forces.

Table 2 represents a summary of the various causes of rotor whirl motion and some of its characteristics as to whirl frequency, the speed at which it occurs, and comments on its general behavior. There are several other mechanisms which have not been included such as electromagnetic effects but this table should represent the major sources of interest in turbomachinery. In general, the whirl behavior and frequency response that the various mechanisms manifest themselves is extremely complex and Table 2 is not to be taken as indicative of the complex behavior.

It is not the intent of this presentation to discuss all of the whirl mechanisms or to present the mathematical foundations of each one of these as this would be prohibitive and beyond the scope of this paper. It is however the purpose of this paper to make the reader aware that whirl motion other than that directly attributed to unbalance exists and that this motion can have dire consequences in high speed turbomachinery if not properly accounted for.

The most serious form of whirl in a turborotor is self-excited motion. This self-excited or nonsynchronous precessive motion is defined as a phenomenon in which the excitation forces inducing the vibration are controlled by the motion. This is in contrast to a forced vibration such as rotor unbalance in which the external excitation is a function of time only. In a forced vibration due to an unbalanced or bowed rotor, the rotor response is synchronous or equal to running speed. In a self-excited whirl, which is often referred to as sustained transient motion, the whirl motion is superimposed upon the running frequency. The superposition of the various frequencies lead to the unusual rotor orbits often observed on high speed rotors. Note that monitoring rotor motion through a synchronous tracking filter such as used in balancing will lose the whirl motion.

An actual turbomachine may experience whirl from one or the interaction of several of the mechanisms listed in Table 2. If a spectrum analysis is taken of a rotor under various conditions of speed and loading, it will usually be observed that some form of whirl motion (nonsynchronous precession) will be present which is superimposed upon the synchronous or unbalanced response motion. It is therefore of considerable importance to the manufacturer and plant operator to be able to identify the occurrence of a potentially dangerous whirl motion that may require shutting down the unit or curtailing its operating conditions of speed or loading. The ultimate objective however of the fundamental understanding of whirl is to design stable turbomachinery that will not encounter self-excited whirl motion throughout its operating speed over a range of loading conditions from no load to surge.

Since the objective of this paper is to survey the problem of rotor-bearing instability, only the major sources of self-excited whirling will be considered.

#### *Hysteretic or Internal Friction Induced Whirling*

At the turn of the century, the majority of the pumps and compressors were reciprocating and the rotating units were massively designed to operate well below the rotor first bending critical speed. The 1920's saw a trend reversing the rotor-design concepts of a previous decade and turbine and particularly compressor and pump manufacturers were beginning to construct lighter weight, higher speed rotors to operate well above the first critical speed.

As more manufacturers went to the flexible rotor design, several encountered severe operating difficulties when operating well above the first critical speed. These problems were at first attributed to the lack of proper balance. In the United States at this time, General Electric encountered a series of failures of blast furnace compressors designed to operate above the first critical speed. These machines were subject to occasional fits of more or less violent vibration of unknown origin. During these disturbances the shaft would vibrate at a low frequency which in some cases could be visually observed. The phenomenon was therefore called by shop men and engineers "shaft whipping." Dr. B. L. Newkirk (40) of the General Electric Research Laboratory was

called in to investigate the nature of the failures. He set up a series of experiments with several units to observe the rotor dynamic behavior. It was observed that at speeds above the first critical speed, these units would enter into a violent whirling in which the rotor centerline precessed at a rate equal to the first critical speed. If the unit rotational speed were increased above its initial whirl speed, the whirl amplitude would increase, leading to eventual rotor failure. To further investigate all aspects and contributing factors to this problem, an experimental test rotor was constructed to simulate a typical compressor unit. Upon extensive testing of this unit, the following important facts were uncovered concerning this phenomenon:

1. The onset speed of whirling or whirl amplitude was unaffected by refinement in rotor balance.

2. Whirling always occurred above the first critical speed, never below it.

3. Whirl threshold speed could vary widely between machines of similar construction.

4. The precession (or whirl) speed was constant regardless of the unit rotational speed.

5. Whirling was encountered only with built-up rotors.

6. Increasing the foundation flexibility would increase the threshold speed of instability.

7. Distortion or misalignment of the bearing housing would increase stability.

8. Introducing damping into the foundation would increase the whirl threshold speed.

9. Increasing the axial thrust bearing load would increase the whirl threshold speed.

10. A small disturbance was sometimes required to initiate the whirl motion in a well balanced rotor.

TABLE 2. CHARACTERISTICS OF ROTOR WHIRL MOTION.

| Cause of Whirl  | Frequency  | Comments  |
|---|--|---|
| Hysteretic or Internal Friction Whirl                       | $N_r > N_1$<br>$N_r \approx N_1$   | Occurs in flexible rotors due primarily to shrink fits and built up parts. Often requires unbalance or initial impulse to start whirl. Violent whirl may occur at first critical speed, disappear and reappear at a higher speed. Realignment of coupling may improve system.   |
| Hydrodynamic fluid film bearings                            | $N_r \leq \frac{N}{2}$ when $\frac{\delta}{C} < 1$<br>(half-frequency whirl)<br>$N_r \approx N_1$ when $\frac{\delta}{C} > 1$<br>$N > 2N_1$<br>(resonant whip) | Often referred to as oil whip, or half frequency whirl. Characteristics change with a flexible shaft where $\delta/C > 1$ and is referred to as resonant whip. Large unbalance may suppress whirl motion.   |
| Aerodynamic Excited Whirl                                   | $N > N_1$<br>$N_r = N_1$   | Whirl motion change with change in power output. May be caused by seals, balance pistons or turbine or compressor tip clearance effects. May not be eliminated by change of unbalance.  |
| Dry Friction Whirl  | $N_r = -n N$   | Usually initiated due to rubbing caused by large unbalance leading to backward whirl motion. Can lead to catastrophic failure on overhung rotor when interacting with disc gyroscopics.   |
| Pulsating Axial Load and Torque Induced Whirl               | $N > N_1$<br>$N_r \approx \left( \frac{1}{2}, 1, \frac{3}{2}, 2 \right) N$   | Occurs in long flexible rotors under large power levels. Pulsating torque may induce large lateral shaft whirling usually equal to $N_1$ .  |
| Asymmetric Shaft Whirl                                      | $N_{1y} < N < N_{1x}$<br>$N_r = \frac{N}{3}, \frac{N}{2}, N$   | Caused by difference in shaft stiffness in two directions. Most violent near first critical speed. Keyways and slots should be avoided. Requires unbalance to initiate motion. May cause difficulty in balancing rotor.   |
| Gyroscopic Induced Whirl                                    | $N_r = \pm n N$  | External excitation or friction rub may excite rotor higher order forward and backward resonance modes. Ball bearing imperfections may initiate backward and forward whirl. Large transient whirl may develop with high acceleration rates with bowed shaft or skewed discs.  |
| Entrained fluid in rotor                                    | $N_1 < N < 2N_1$<br>$N_r = N_1$  | Instability encountered in high speed centrifuges. May occur in any high speed turbine or compressor if liquid, oil or steam condensate becomes inadvertently trapped in the internal cavity of a hollow rotor. Also produces large unbalance vibrations and is impossible to balance.  |
| Subharmonic and Superharmonic Whirl Induced by Nonlinearity | $N_r = N_1, 2N_1, N_2,$<br>$N = \frac{1}{3} N_1, \frac{1}{2} N_1, 2N_1, N_2$   | Requires unbalance or external excitation to initiate and appears frequently in under damped rotors. May combine with gravity in heavy horizontal rotor to produce a secondary critical speed. Half frequency whirl motion produced often confused with oil film whirl. Effect may be often alleviated by rotor balance, alignment of bearings or coupling. |

It became clear to Newkirk that the rotor dynamic behavior could not be attributed to a critical speed resonance, since the high vibrations encountered always occurred above the first critical speed and refinement of balance had no effect upon diminishing the whirl amplitudes. There was nothing in the literature at that time to indicate that any mode of motion, other than synchronous whirl, was possible. During the course of the investigation, a theory of the cause of the vibration was postulated by A. L. Kimball (26). Kimball suggested that forces normal to the plane of the deflected rotor could be produced by the hysteresis of the metal undergoing alternate stress reversal cycles. Newkirk concluded that these out-of-phase forces could also be developed by a disk shrunk on a shaft. Newkirk was unable at the time to explain why increased bearing or foundation flexibility would improve stability.

Newkirk concluded that the internal friction created by shrink fits of the impellers and spacers was the predominate cause of the observed whirl instability. He had observed that when all shrink fits were removed from the experimental rotor, no whirl instability could develop.

Kimball (26), at Newkirk's suggestion, constructed a special test rotor with rings on hubs shrunk on the shaft. He did indeed confirm Newkirk's conclusion that the frictional effect of shrink fits is a more active cause of shaft whirling than the internal friction within the shaft itself. Measurements showed that, even with the rather light shrinkages used in the tests, the effective internal friction may be increased from two to five times its original value. In fact, Kimball found that long clamping fits always lead to trouble with high speed rotors.

For the case of a hub or a sleeve which is fastened to a shaft which is afterward deflected, either the surface fibers of the shaft must slip inside the sleeve as they alternately elongate or contract, or the sleeve itself must bend along with the shaft. Usually both actions occur simultaneously to an extent which depends upon the tightness of the shrink fit, and the relative stiffness of the two parts. H. D. Taylor (41), after conducting numerous tests with various hub configurations, concluded that the axial contact length of shrink fits should be as short as permissible and as tight as possible without exceeding the yield strength of the material.

Robertson (49) reports that even short, highly stressed shrink fits are not entirely devoid of the problem. He states that even small, tight shrink fits may develop whirl instability, provided the rotor is given a sufficiently large initial disturbance or displacement to initiate relative internal slippage in the fit. If long shrink fits such as compressor wheels and impeller spacers are employed, it is important that these components be undercut along the central region of the inner bore so that the contact area is restricted to the ends of the shrink fit. Robertson (49) shows several designs of hubs and bosses which have been found to be beneficial in reducing internal friction effects.

Robertson also concludes that a similar effect can be produced by any friction which opposes a change of the deflection of the shaft, such as friction which exists

at the connections of flexible couplings, and gear type couplings. He refers to this group of friction forces as hysteretic forces.

Following the analysis of Smith (52), the equations of motion of a single mass rotor on damped elastic supports are given as follows:

$$m\ddot{x} + (u + v)\dot{x} + \omega\sqrt{uv}y + Kx = me\omega^2\cos\omega t \quad (1)$$

$$m\ddot{y} + (u + v)\dot{y} - \omega\sqrt{uv}x + Ky = me\omega^2\sin\omega t \quad (2)$$

where

$$v = C_b \left( \frac{K_s}{K_b + K_s} \right)^2$$

is the stationary damping coefficient representing the effect of damping in the bearing supports

$$u = C_i \left( \frac{K_b}{K_b + K_s} \right)$$

is the stationary damping coefficient representing the effect of damping in the bearing supports

Note that the rotor equations of motion are cross-coupled by the influence of internal friction. When there is stationary damping but no internal friction damping, the equations of motion are uncoupled and the free or transient motion is a damped oscillatory behavior and is always stable.

It can be shown that the threshold of stability speed for this simple system is given by the following relationship (17).

$$\omega_s = \omega_c \left[ 1 + \left( \frac{K_s}{K_b} \right)^2 \left( \frac{C_b}{C_i} \right) \right] \quad (3)$$

Hence it is concluded that a system with internal damping but no external damping is unstable at all speeds above the critical speed  $\omega_c$ . An analysis of the precessive motion shows that the whirl rate is approximately equal to the rotor system critical speed  $\omega_c$ . An investigation of the unbalance response of the system indicates that external damping restricts the unbalance response amplitude but the internal damping does not.

Figure 3 represents the dimensionless rotor stability threshold vs. support flexibility coefficient  $R$ . Figure 3 shows that in general the rotor stability threshold is always equal or greater than the system critical speed and is a function of the internal damping, support damping, and stiffness coefficients. Note that if no damping is introduced into the foundation the stability criteria reduces to

$$\omega_s = \omega_c \quad (4)$$

The above relationship implies that if bearing flexibility is incorporated into the system without bearing damping, then the system critical speed and the whirl threshold speed will be reduced. This criteria therefore is inadequate to explain the observations of Newkirk that greater stability can be achieved by foundation flexibility only. The stability criteria for a symmetric foundation states that both foundation flexibility and damping must be included to increase rotor stability. Smith

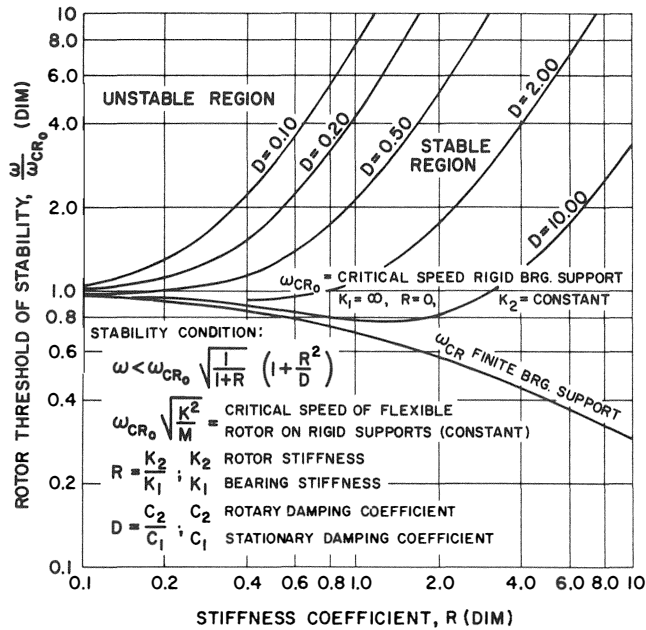


Figure 3. Stability Threshold of a Flexible Rotor With Internal Friction on a Symmetric Elastic Bearing Support.

(52), Gunter (20), and Kellenberger (25) have demonstrated that it was the influence of anisotropic bearing stiffness that was responsible for the improved stability observed by Newkirk even in the absence of damping in the bearings.

When the stability threshold is exceeded as shown in Figure 4a for the linear system as represented by Equations 1 and 2, the rotor motion becomes unbounded and grows exponentially with time. In an actual system the rotor orbit forms a limit cycle due to the presence of nonlinear effects either in the shaft or the bearings as shown in Figures 4 b and c. Various authors such as Tondl (59) and Dimentberg (13) have chosen considerably more complex models in which the internal damping is independent of frequency, nonlinear or amplitude dependent.

Figure 5 represents the stability criteria of Tondl and represents the rotor external damping  $D$  and the stability parameter  $H(Z)$  versus frequency ratio  $\nu$ . The function  $H(Z)$  is a function of the internal rotor friction, external damping and rotor unbalance response. The rotor is unstable when the  $D^2$  and  $H(Z)$  curves intersect. This figure demonstrates the important conclusion that if a rotor becomes unstable just above the critical speed that it will not necessarily be unstable at all higher speeds. For light damping the rotor is unstable above the critical speed. As the damping is increased the rotor is unstable only for a very narrow range above the critical speed from the speed ratio of  $\nu_2$  to  $\nu_3$ . The rotor restabilizes above the critical speed and is stable until the rotor speed is increased to a much higher speed  $\nu_4$  where instability is again encountered.

The stability criteria of Tondl may well explain the unusual rotor whirl motion reported by Stodola (56) over 40 years ago in which certain gear type couplings would cause large nonsynchronous whirl motion over a short operating speed range.

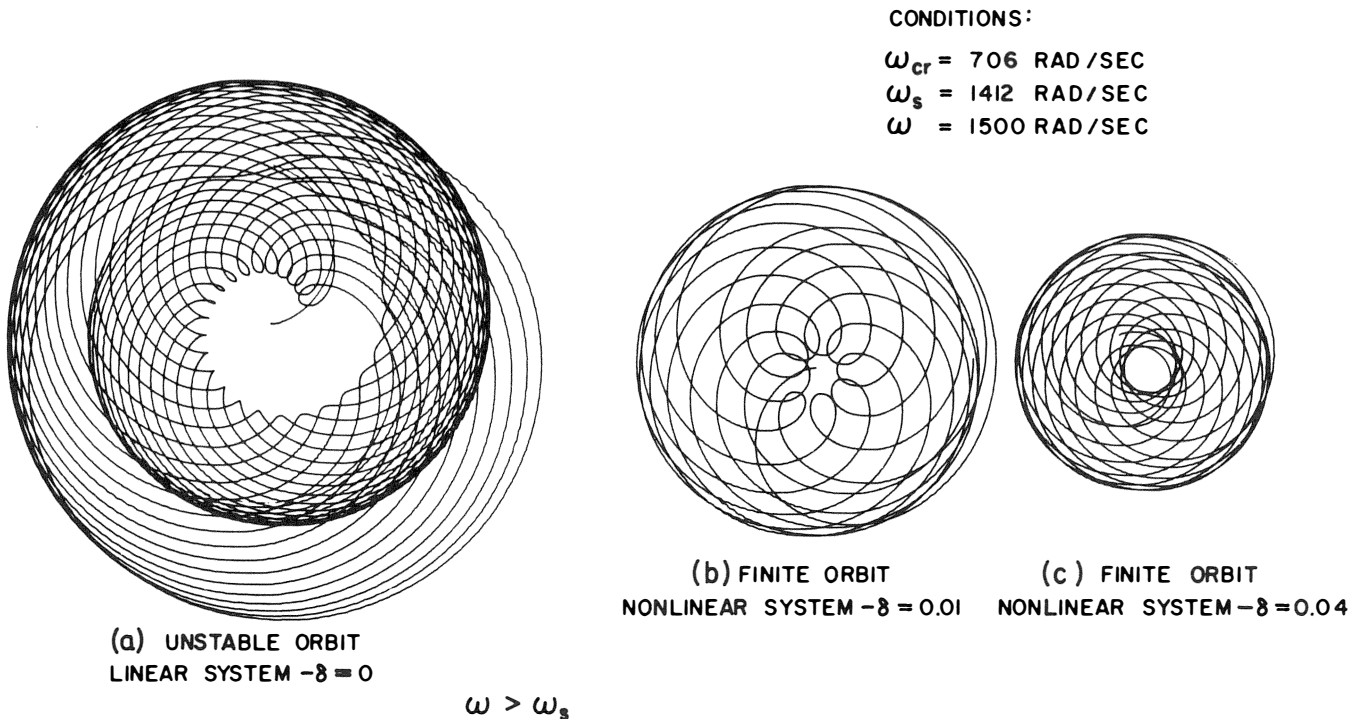


Figure 4. Effect of Nonlinearity on Rotor Motion Above the Threshold of Stability.

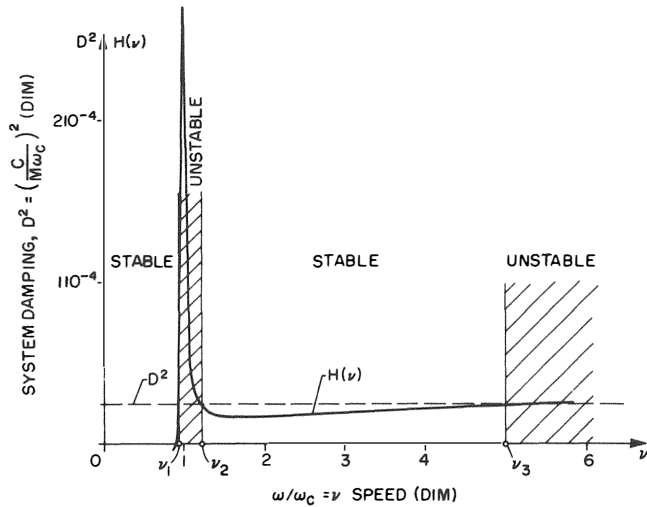


Figure 5. Rotor Stability With Internal Damping (Ref. Tondl).

An extensive investigation of the self excited whirl motion of high speed textile spindles was conducted by Kushul' (17). The spindles were composed of a built

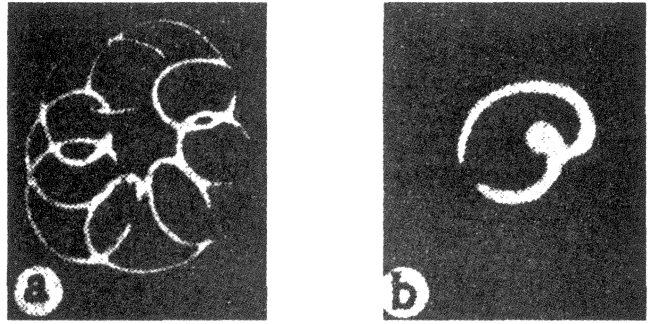


Figure 6. Photographs of Rotor Motion With Internal Friction (Ref. Kushul').

up structure of a long wooden spindle inserted over a thin steel shaft. It is easy to visualize how such a long shrink fit could lead to stability problems. Robertson reported in 1935 (49) that long continuous shrink fits invariably lead to difficulties when operating above the rotor first critical speed.

Figure 6 represents typical rotor orbits obtained by Kushul' on the textile spindles operating above the stability threshold. At the time Kushul' recorded this data,

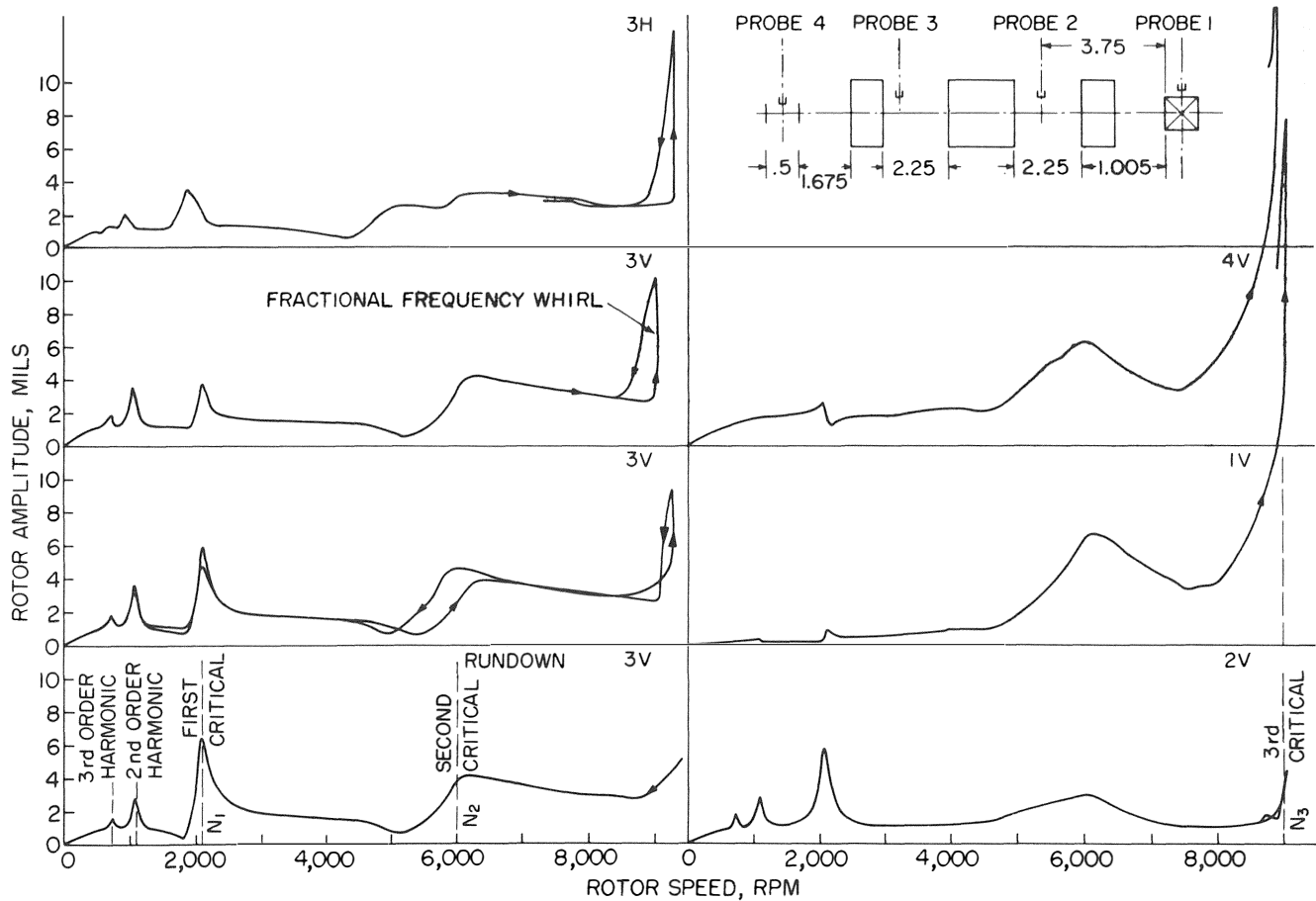


Figure 7. Unbalance Response and Self-Excited Instability With a Three-Mass Rotor System.

precision electronic proximity probes to monitor the rotor motion were not available so he had to resort to an optical system. He attached a fine needle to the spindle end and obtained the following pictures by photographing the resulting motion under a microscope. Figure 6a represents the spindle whirl motion at the stability threshold. As the rotational speed was increased, Kushul' observed that the motion could abruptly change to the orbit as shown in Figure 6b in which the whirl rate is  $\frac{1}{2}$  of rotational speed. This behavior cannot be obtained in a linear system.

Figure 7 represents the unbalance response and self-excited whirl instability encountered with a three-mass rotor system due to internal rotor friction. The rotor was run from 0 to 10,000 RPM and rotor traces of the motion were obtained at the different probe locations along the shaft in the horizontal and vertical direction. The rotor has three critical speeds in the operating range at approximately 2,000, 6,000 and 10,000 RPM. Note that below the first critical speed super harmonic oscillations are encountered due to non-linearity in the rotor and bearing supports. Superharmonic oscillations are often experienced in machinery and are usually not of serious consequences.

The critical speeds of this particular model are similar to those shown in Figure 8. The first is a symmetric bending of the shaft, the second in a conical mode and the third mode is a symmetric bending in which the amplitude at the bearing locations is out of phase to the amplitude at the center shaft and the maximum amplitude

is experienced at the bearings. It is of interest to note that a fourth critical speed is encountered at a much higher speed with the three mass model. This particular mode is caused by the gyroscopic effects of the disks. Figure 7 shows that as the rotor approaches the third critical speed, the maximum amplitude is experienced at probe locations 4 and 1 which correspond to the bearing ends. The motion increases rapidly due to rotor unbalance and is synchronous. Probe 2V near the rotor center shows little increase in amplitude as it approaches the third critical speed. Abruptly the center of the shaft at station 3 jumps into a violent self-excited whirl instability. Destruction of the rotor would have resulted if the unit were not immediately shut down. The mechanism that caused this instability is obviously more complicated than the simplified systems described by equation 1 and 2 and is more closely simulated by the non-linear system of Tondl (60). Although unbalance is normally not a major influence in rotor instability, it was seen in this case that large unbalance was necessary to initiate the whirl instability. This observation is similar to the earlier conclusions of Newkirk (40) on the whirl motion encountered with centrifugal compressors due to the internal friction of shrink fits.

Figure 9 represents the whirl orbits of an unbalanced rotor with internal friction on anisotropic supports. The rotor is similar to the configuration represented in Figure 6 for the linear case except that the support stiffness in the X direction has been reduced to  $\frac{1}{2}$  its original value. This has caused the instability speed to increase from 1412 rad/sec to 3230 rad/sec.

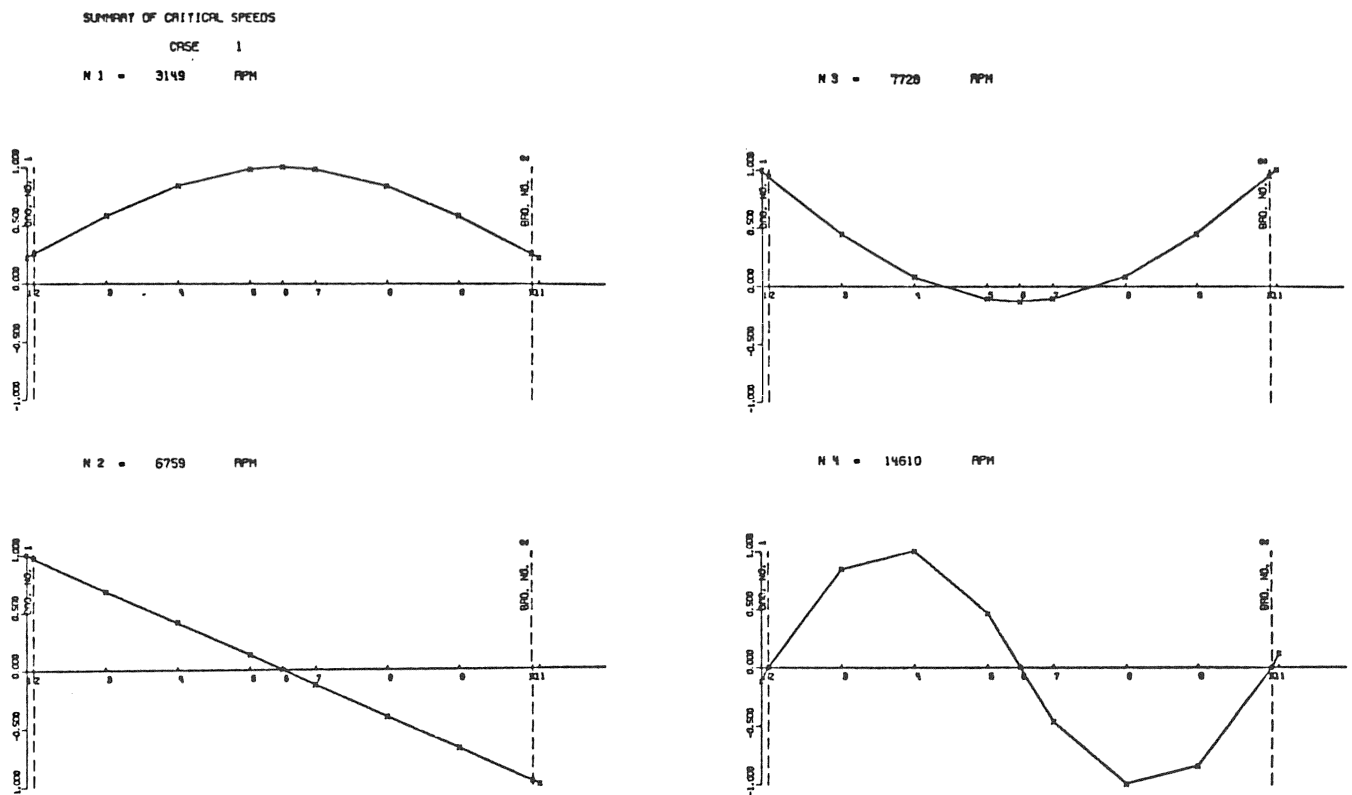


Figure 8. Critical Speeds and Mode Shapes for a Three-Mass Rotor Including Gyroscopics.



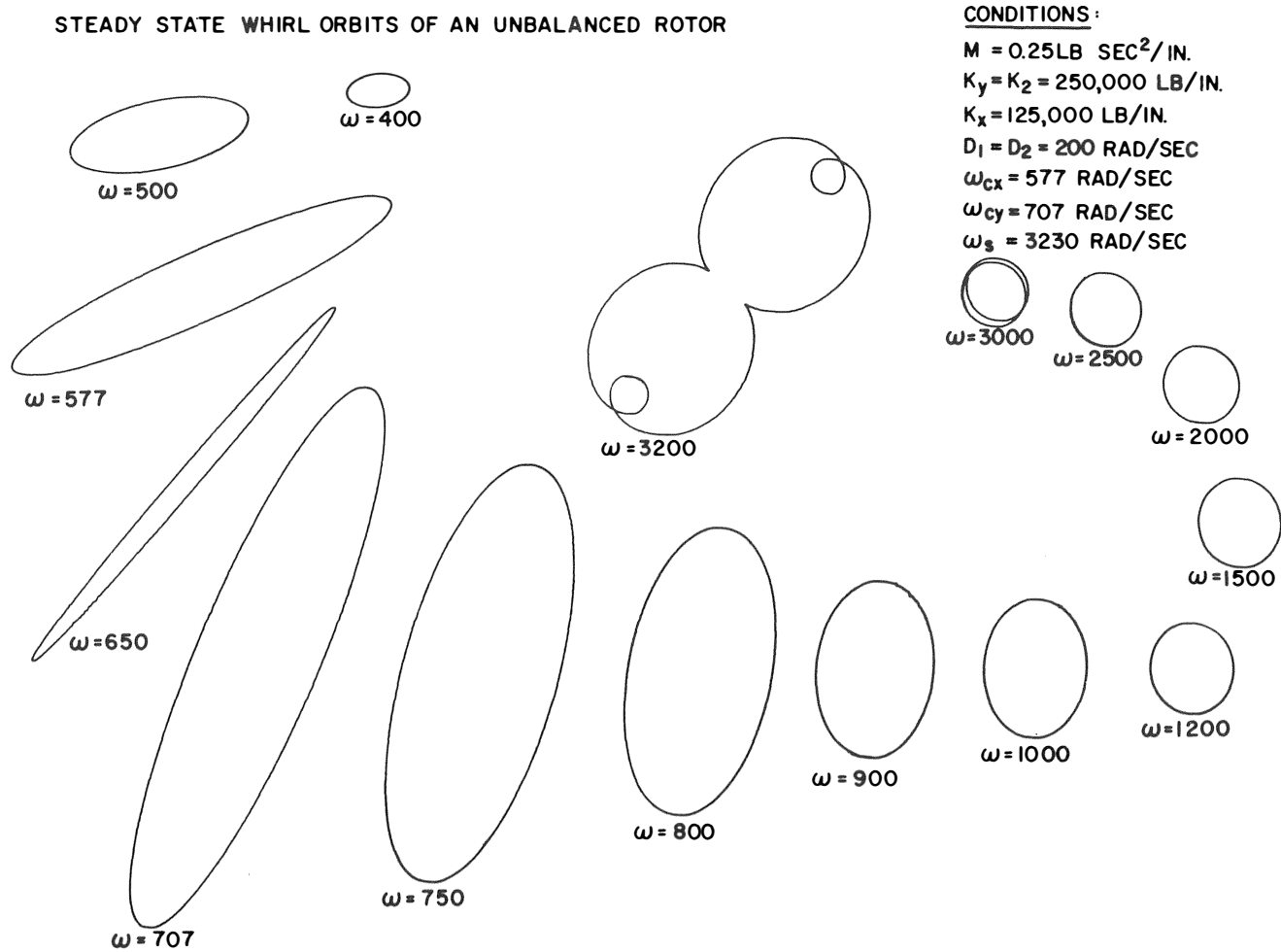


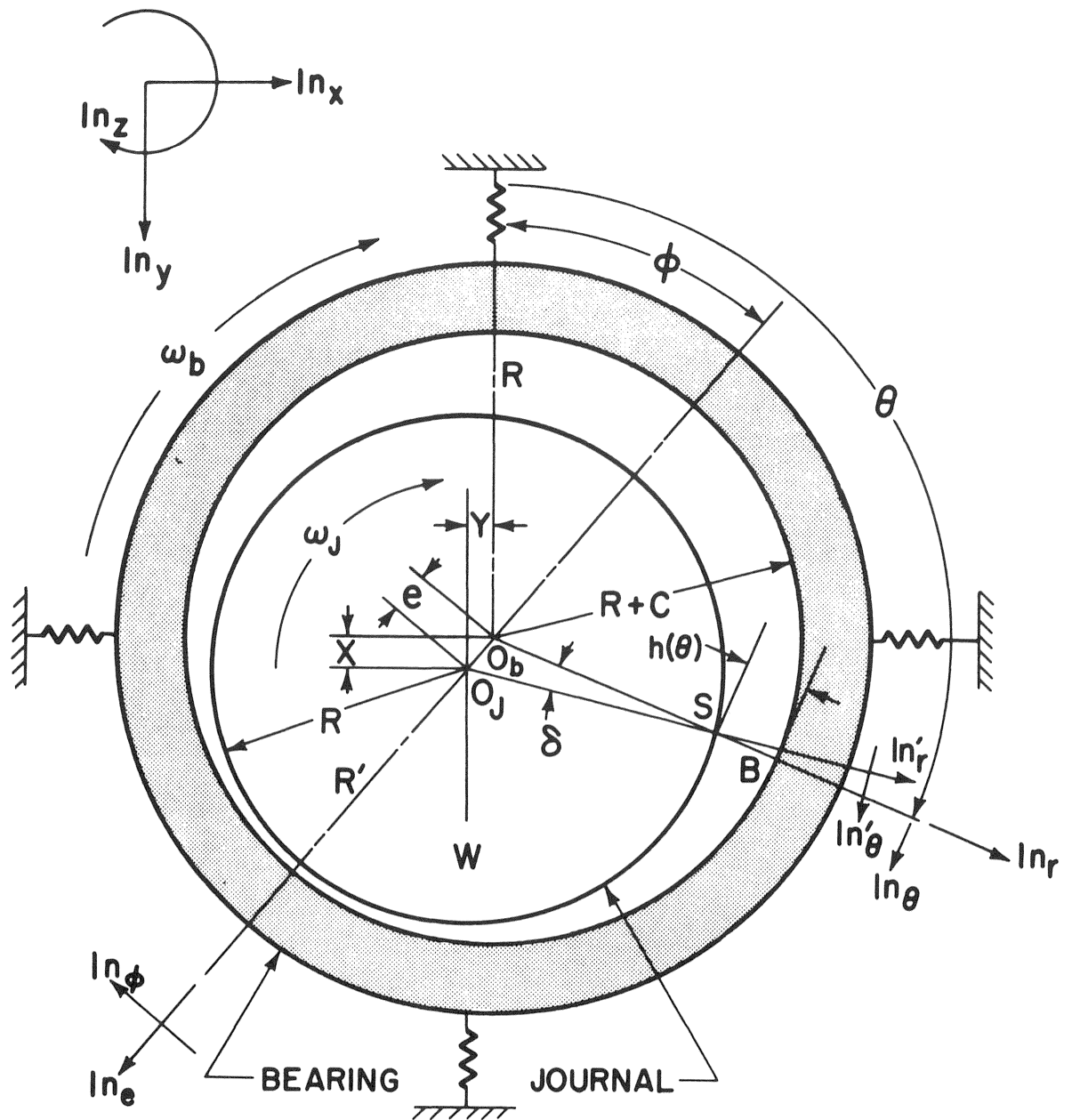
Figure 9. Whirl Orbits of an Unbalanced Rotor With Internal Friction on Anisotropic Supports.

Thus it is seen with certain cases of instability the threshold speed may be improved by supporting the rotor on an anisotropic supports. When the bearing characteristics are unsymmetrical the synchronous or running speed orbits due to unbalance are not circular but are elliptical. The system has two critical speeds; one corresponding to the X direction and one for the Y direction. When the rotor is operating at the speed corresponding to the horizontal critical speed the ellipse is oriented predominantly along the horizontal axis. As the speed is increased the ellipse changes from a horizontal to a vertical orientation. When the speed is increased above the critical speeds, the orbit changes to circular orbit with a radius equal to the unbalance eccentricity. Upon approaching the stability threshold at 3200 rad/sec the transient motion no longer damps out but grows rapidly. Even when the rotor is below the stability threshold at 3,000 rad/sec there is a small component of non-synchronous whirl motion present in the orbit. It is often observed in turbomachinery that a small component of whirl motion exists before the threshold of stability is encountered.

#### Fluid Film Bearing Whirl

One of the major sources of whirl instability in turbomachinery is that caused by fluid film bearings and seals. Figure 10 represents an idealized hydrodynamic bearing configuration. Under normal operating conditions the shaft and the bearing surface are separated by a film of oil. The rotation of the shaft in the converging film area builds up a pressure profile which supports the shaft as given by the following well-known Reynolds equation.

$$\begin{aligned}
 & \frac{\partial}{\partial \Theta} \left[ H^3 \rho \frac{\partial \rho}{\partial \Theta} \right] + \\
 & \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial Z} \left[ H^3 \rho \frac{\partial \rho}{\partial Z} \right] \\
 & = \Lambda \left( \left[ 1 - \frac{2\Theta}{\omega} \right] \frac{\partial}{\partial \Theta} \left[ \rho H \right] \right. \\
 & \quad \left. + 2 \frac{\partial}{\partial Z} \left[ \rho H \right] \right) \quad (5)
 \end{aligned}$$



## BEARING GEOMETRY

Figure 10. Hydrodynamic Journal Bearing Configuration.

In addition to the bearings, there are other components in a typical turbomachine such as the seals in which the shaft and stationary seal or casing is separated by a film of fluid. In this case the seal can generate a hydrodynamic pressure distribution around the shaft and act as a bearing. The forces induced on a shaft may create a condition known as oil film or half-frequency rotor whirl. For example Figure 11 represents a typical pressure distribution developed in a hydrodynamic journal bearing for a particular eccentricity. In this case the

film is considered to cavitate when the pressure distribution is below the oil vapor pressure. If the film does not cavitate in a plain journal bearing then the bearing force-displacement attitude angle is  $90^\circ$  and the system will be unstable at all speeds.

Figure 12 represents the whirl motion of a vertical balanced rotor running at 6,500 RPM. The rotor is given a small initial displacement and it orbits outward at a precession rate which is approximately  $\frac{1}{2}$  the rotational speed. If the rotor were permitted to run in this

condition the bearing would eventually fail similar to the bearing shown in Figure 13. Figure 13 is a photograph of a journal bearing that was operated above the stability threshold and shows failure due to massive fatigue pitting. The large orbit of the shaft caused a rotating pressure profile similar to that shown in Figure 11 which eventually caused massive fatigue pitting of the journal surface. It is obvious that the large orbiting represented in Figure 12 would be unacceptable for continuous operation.

Figure 14 represents the whirl motion of the vertical rotor with an unbalance added to the shaft. The unbalance eccentricity ratio  $EMU = 20\%$  of the bearing radial clearance. Note that when the rotating unbalance force has been added to the rotor the orbit is smaller. The orbit is now bounded and forms a limit cycle due to the nonlinearity of the film. Hence it is seen that for the case of the vertical rotor, the addition of unbalance actually reduces the rotor whirl orbit. There have been numerous incidents where manufacturers of water lubricated vertical pumps have used plain journal bearings satisfactorily. This may be explained by the effect of unbalance on the rotor motion and also the effect

of bearing misalignment. Various investigators have seen that slight misalignment of the bearing housing will promote stability. The plain journal bearing however is a poor choice for use in a vertical rotor-bearing system.

It is of interest to note that investigations with gas bearings for gyroscopes have shown that quite often the manufacturer would produce a bearing with lower stability characteristics as the quality control would increase. As more care was made in producing a well aligned rotor-bearing and a circular geometry, the stability characteristics of the system would decrease. Such effects as bearing misalignment, ellipticity, and surface irregularities have been known to change the bearing stability characteristics.

The orbit as shown in Figure 14 represents a combination of synchronous motion due to unbalance and half frequency whirl motion due to the bearing fluid film. By examination of the size of the inner lobe with respect to the orbit, the ratio of the synchronous to the nonsynchronous whirl component can be estimated. For example Figure 15 represents various combinations of

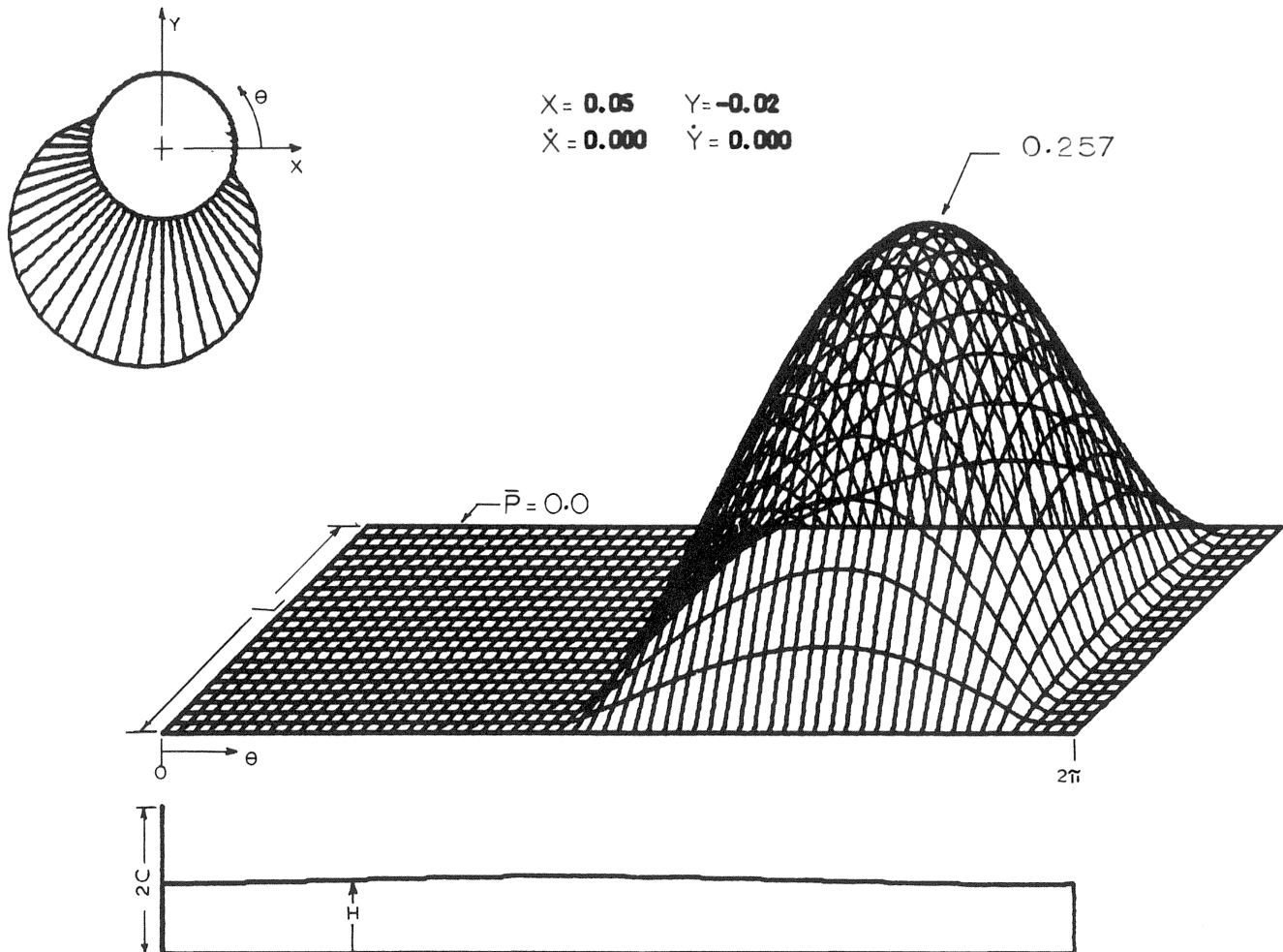


Figure 11. Journal Bearing Pressure Distribution Considering Cavitation.

## VERTICAL BALANCED ROTOR

NO. 111189

N = 6500 RPM  
 A = 1.00 IN.  
 L = 1.00 IN.  
 C = 5.00 MILS  
 TASMAY = 1.39  
 S = 0.048  
 SS = 0.012  
 $\dot{X}(T=0) = 0.1$   
 WT = 0.00  
 W = 1800 LB.  
 MU<sub>W</sub> = 1.000 REYNS  
 FMAX = 2493.0 LB. AND  
 OCCURS AT 4.96 CYCLE  
 WS = 2.45  
 ES = 0.814

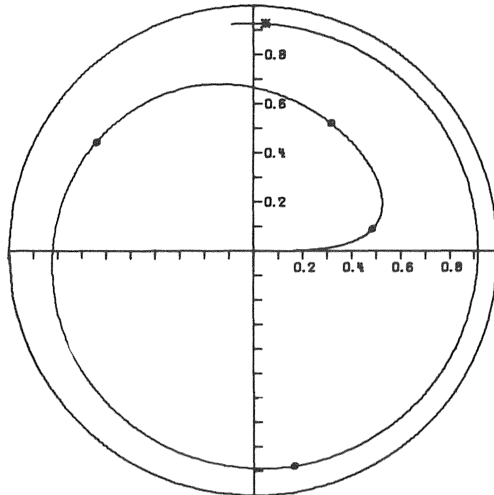


Figure 12. Journal Orbit of a Balanced Vertical Rotor With Small Initial Velocity for 5 Cycles.

synchronous and half frequency whirl. If a small component of half frequency whirl motion is superimposed upon the synchronous motion then the circular orbit begins to appear as two circles, one superimposed upon the other. As the half-frequency whirl component increases, the size of the inner lobe decreases. A comparison of the figures in Figure 15 to the orbit in Figure 14 indicates that the half-frequency whirl motion is

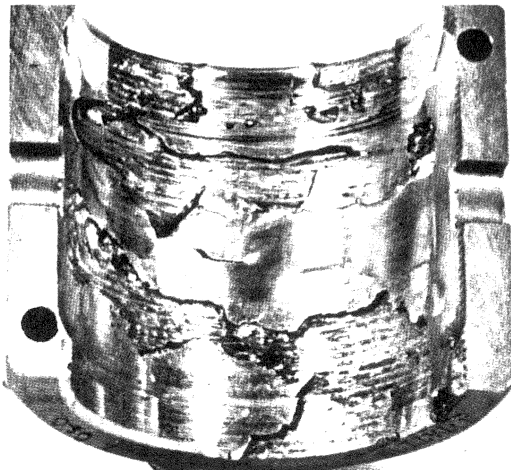


Figure 13. Photograph of Failed Journal Bearing Operated Above the Stability Threshold.

## VERTICAL UNBALANCED ROTOR

NO. 41593

N = 6500 RPM  
 A = 1.00 IN.  
 L = 1.00 IN.  
 C = 5.00 MILS  
 TASMAY = 1.31  
 S = 1.733  
 SS = 0.433  
 EMU = 0.20  
 SU = 1.446  
 TADMAY = 1.10  
 WT = 0.00  
 W = 50 LB.  
 MU<sub>W</sub> = 1.000 REYNS  
 FMAX = 65.7 LB. AND  
 OCCURS AT 0.58 CYCLE  
 WS = 2.45  
 ES = 0.211  
 FU = 59.95 LB.  
 FUATIO = 1.20  
 ESU = 0.244

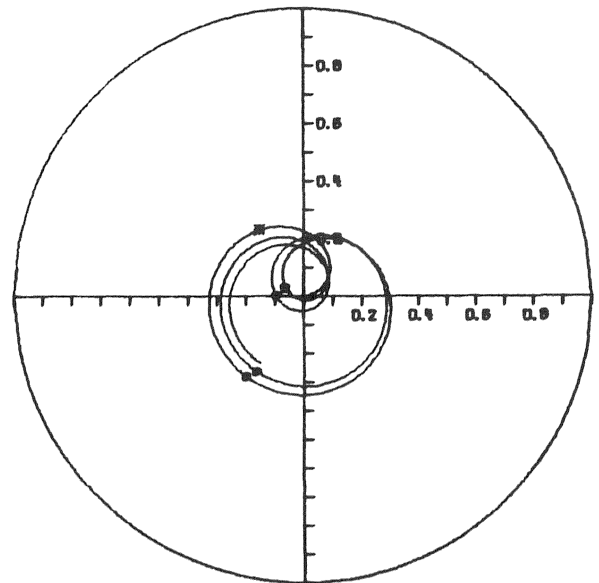


Figure 14. Journal Orbit of an Unbalanced Vertical Rotor for 5 Cycles ( $N = 6500$ ,  $Emu = 0.2$ ) With Damped Half-Frequency Whirl.

approximately equal to the synchronous unbalance component [ $A=B=.5$ ].

Figure 16 represents the stability threshold based on the short journal approximation considering various values of external loading. For example the line  $Wt = 1$  represents the horizontal rotor. Values above the threshold line will be unstable and below the line will be stable. For example the stability contour for the horizontal rotor shows that if the bearing eccentricity is above .75 the system will be stable for all speeds. Conversely if the stability parameter  $\omega_s$  is less than 2.5 the bearing will be stable for all values of eccentricity. As the value of the external loading parameter  $Wt$  decreases, the stability boundaries of the system decreases. The value of  $Wt = 0$  represents the vertical rotor and the system will be unstable for all speeds and eccentricities.

Figure 17 represents the orbit of a balanced horizontal rotor at the stability threshold. The rotor is released from the origin and precesses at approximately  $\frac{1}{2}$  frequency whirl in a diminishing orbit until it reaches the steady state eccentricity of 0.2. The bearing steady state load-deflection attitude angle is about  $70^\circ$ . If the rotor speed is now increased from 6,500 to 10,000 RPM,

the stability parameters  $\omega_s$  increases from 2.45 to 4.0. According to the stability plot of Figure 16 the rotor is well above the stability threshold and hence will be unstable. Figure 18 shows that the rotor is highly unstable and precesses outward with approximately  $\frac{1}{2}$  frequency whirl motion. The orbit will continue to grow until it reaches an eccentricity of approximately .7 in which it then forms a limit cycle. Although the bearing does not immediately fail, the large whirl motion is undesirable because of the possibility of fatigue pitting as illustrated in the bearing of Figure 13.

If, however, unbalance equivalent to 20% of the bearing clearance is introduced the whirl orbit is con-

siderably reduced from that experienced with the balanced rotor as shown in Figure 19. This however is an undesirable practice because of the high transmitted forces. For example the rotating unbalance load is 150 lbs. while the maximum force transmitted is 213 lbs. Thus the dynamic transmissibility is greater than 1. In a properly designed rotor-bearing support system, the dynamic transmissibility should be less than 1 to achieve good attenuation of the forces transmitted through the structure.

The stability characteristics of the circular journal bearing are rather limited and therefore are not applicable to modern high speed turbo-machinery because of the

### ANALOG COMPUTER TRACES OF VARIOUS COMBINATIONS OF SYNCHRONOUS AND HALF-FREQUENCY WHIRL

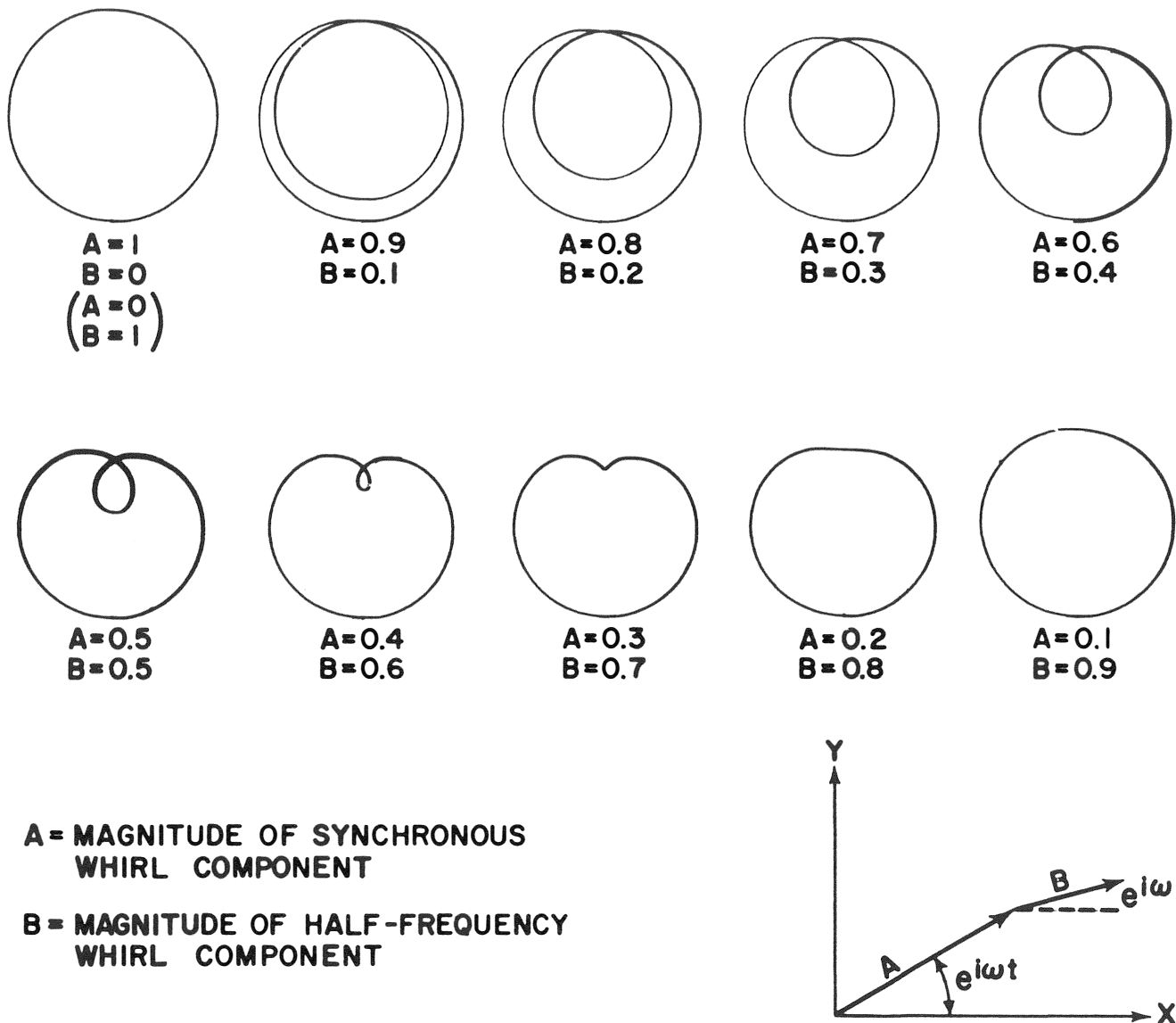


Figure 15. Analog Computer Traces of Various Components of Synchronous and Half-Frequency Whirl.

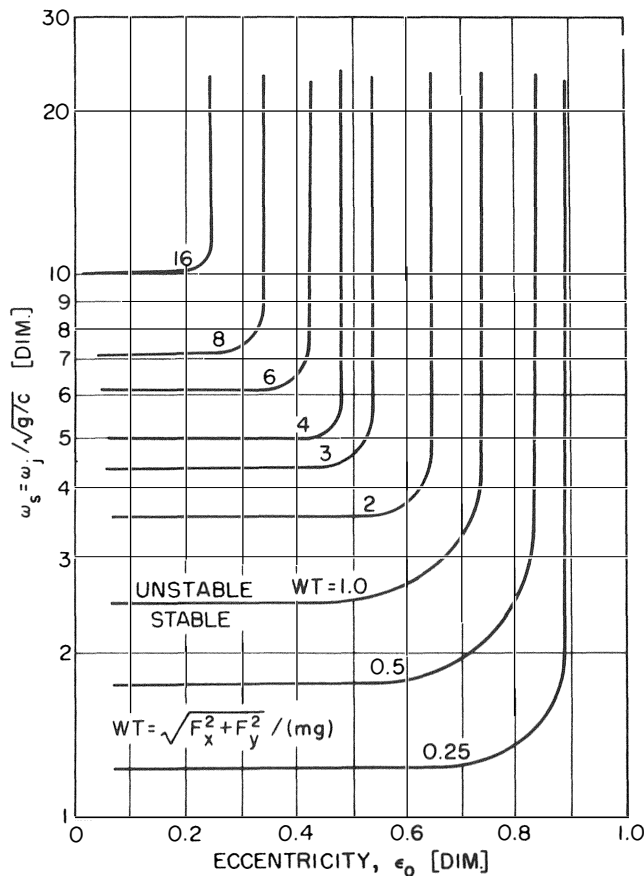


Figure 16. Stability Map for the Short Journal Bearing Considering Constant Loading.

possibility of self-excited whirling. One of the ways of improving the stability of the plain journal bearing is by means of the addition of a pressure dam to preload the bearing. It can be seen from the stability chart of Figure 16 that if the load parameter  $Wt$  is greater than 1 then the stability characteristics of the bearing will improve. For example with a  $Wt$  value of 6 the bearing will be stable for all eccentricity above .45 and for speeds below  $\omega_s = 6$ .

#### Example 1

Consider the following sample calculation of the stability characteristics of a two stage centrifugal compressor on water lubricated pressure dam bearings. The bearing characteristics are as follows:

$$\begin{aligned} D &= 2.5 \text{ in.} & L &= 0.75 \text{ in.} \\ C_r &= 0.0015 \text{ in.} & W/\text{brg} &= 50 \text{ lb.} \\ M &= 1.0 \times 10^{-7} & N &= 19,100 \text{ RPM.} \\ \text{Pocket Depth} &= 0.031 \text{ in.} & \omega &= 2000 \\ & & \text{Preload} &= 100 \text{ lb.} \end{aligned}$$

$$\omega_g = \frac{g}{C} = 507, \quad \frac{\omega}{\omega_g} = 3.94, \quad Wt = 3.00$$

From Figure 16 assuming  $\epsilon < 0.5$  for  $Wt = 3.00$ ,  $\omega_s = 4.3$  Threshold Value since  $\omega_s > \omega/\omega_g$  system is marginally stable.

There are other ways to improve bearing stability other than the pressure dam bearing such as the externally pressurized bearing, tilting pad, or the multilobe bearing.

Figure 20 represents the motion of an externally pressurized gas bearing grinder spindle operating at 52,500 RPM. In Figure 20a the spindle shows only a small synchronous orbit of  $180 \mu$  in. With only a small change in the operating speed, the spindle developed the large whirl orbit as shown in Figure 20b. Further increase in speed beyond this point would have resulted in destruction of the grinder spindle. Extreme care must be exercised in operating gas bearing machinery above the stability threshold. Applications of gas bearings to small high speed turbomachinery are now being developed using externally pressurized, tilting pad, and foil bearings.

The multilobed bearing configuration as shown in Figure 21 has superior stability characteristics as compared to the plain journal bearing. Figure 22 represents the stability of the three lobed bearing configuration for various values of preload. A zero preload represents an axial groove bearing. It is of interest to note that

#### BALANCED ROTOR

NO.102681

$N = 6505 \text{ RPM}$

|                         |  |
|-------------------------|--|
| $A = 1.00 \text{ IN.}$  | $W = 47 \text{ LB.}$                   |
| $L = 1.00 \text{ IN.}$  | $MUS = 1.000 \text{ REYNS}$            |
| $C = 5.00 \text{ MILS}$ | $FMAX = 59.9 \text{ LB. AND}$          |
| $TASMAX = 1.27$         | $\text{OCCURS AT } 0.53 \text{ CYCLE}$ |
| $S = 1.845$             | $WS = 2.45$                            |
| $SS = 1.845$            | $ES = 0.200$                           |

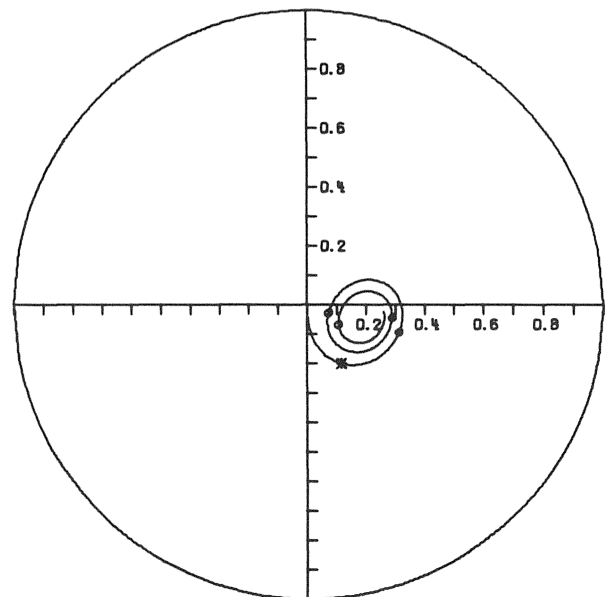


Figure 17. Journal Orbit of a Balanced Horizontal Rotor at the Stability Threshold ( $N = 6500$ ,  $W = 50$ ,  $C = 0.005$ ,  $L/D = 1/2$ ).

## BALANCED ROTOR

NO. 102684

N = 10621 RPM

|               |                               |
|---------------|-------------------------------|
| A = 1.00 IN.  | W = 47 LB.                    |
| L = 1.00 IN.  | MU <sub>S</sub> = 1.000 REYNS |
| C = 5.00 MILS | FMAX = 156.2 LB. AND          |
| TASMAX = 3.32 | OCCURS AT 5.00 CYCLE          |
| S = 3.013     | WS = 4.00                     |
| SS = 0.753    | ES = 0.130                    |

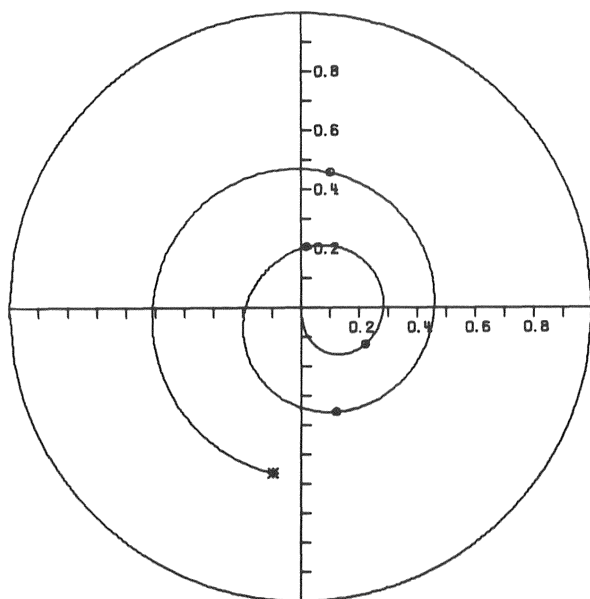


Figure 18. Journal Orbit of a Balanced Horizontal Rotor Above the Stability Threshold ( $N = 10,621$ ,  $W = 47$ ,  $C = 0.005$ ).

the axial groove bearing does not appear to have superior stability characteristics to the plain journal bearing. Although improvements in stability have been experienced with the axial groove bearing in comparison to the plain journal bearing, it is felt that this effect may be due to the misalignment. Note that the stability threshold of the multilobe bearing with a preload factor of  $\delta = 0.6$  is  $W_s = 9$  which is over twice the stability value of the pressure dam bearing in the previous example. In the vertical position, the axial groove bearing ( $\delta = 0$ ) is completely unstable. The stability threshold of the optimum three-lobe bearing of  $L/D = 1.0$ , with an offset factor  $\delta = 1.0$  (completely converging film) and preload of 0.5 is given by

$$N_{RPM} = \frac{300\mu L}{M} \left( \frac{R}{C} \right)^3$$

Improved stability is obtained by means of a flexible support or a floating bush damper as shown in Figure 23. Figure 23 represents the stability threshold vs. the bush clearance for a particular rotor configuration. Note that a maximum stability threshold for the plain journal bearing is 2.5. If, however, the bush clearance is made

too large or too small then there is little improvement in stability.

The effect of rotor flexibility can have a pronounced effect on the threshold speed and the oil film whirl behavior in a rotor as shown by various investigators such as Hori (24), Lund (35), Ruhl (50) and others. For example Figure 24 represents the stability characteristics and amplitude and frequency measurements obtained by Hori with a flexible rotor in a journal bearing. In Hori's figure the stability threshold is over twice the rotor first critical speed as shown in Figure 24 a, b and c. Note also that the rotor amplitude does not become unbounded above the threshold speed but forms a limit cycle.

## Aerodynamic Induced Whirling

In addition to stability problems created by hydrodynamic bearings in turbomachinery another important cause of instability is aerodynamic cross coupling forces and instability created by labyrinth seals and balance pistons. Stodola, in his earlier work on steam turbines, reported on the influence of leakage in creating instability problems in steam turbines. Alford has reported

## UNBALANCED ROTOR

NO. 102685

N = 10621 RPM

|               |                               |
|---------------|-------------------------------|
| A = 1.00 IN.  | W = 47 LB.                    |
| L = 1.00 IN.  | MU <sub>S</sub> = 1.000 REYNS |
| C = 5.00 MILS | FMAX = 213.8 LB. AND          |
| TASMAX = 4.55 | OCCURS AT 0.93 CYCLE          |
| S = 3.013     | WS = 4.00                     |
| SS = 0.753    | ES = 0.130                    |
| EMU = 0.20    | FU = 150.46 LB.               |
| SU = 0.941    | FURATIO = 3.20                |
| TADMAX = 1.42 | ESU = 0.331                   |

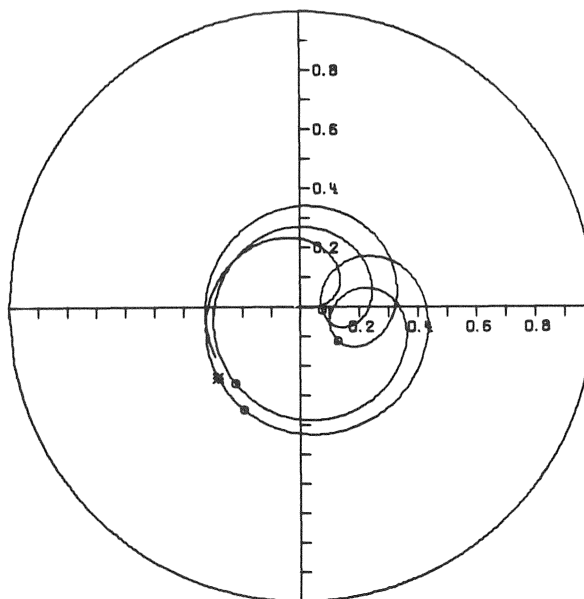


Figure 19. Journal Orbit of an Unbalanced Horizontal Rotor Above the Stability Threshold ( $N = 10,621$  RPM,  $W = 47$ ,  $C = 0.005$ ,  $Emu = 0.20$ ).

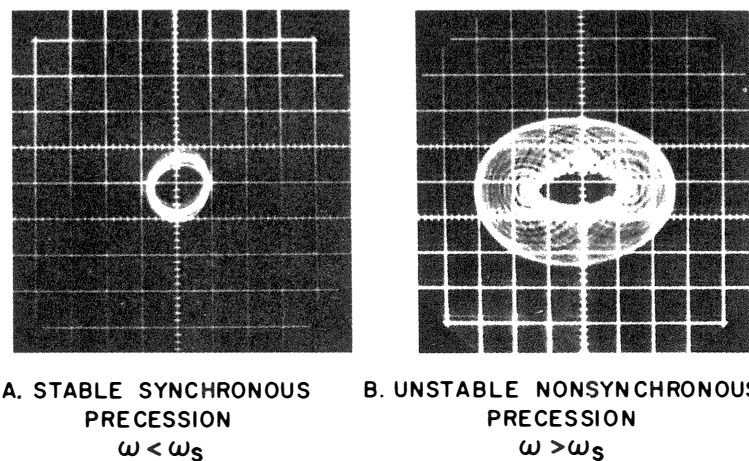
(1) on aerodynamic cross coupling effects in causing stability problems in jet aircraft engines. Although a centrifugal compressor may be designed to have a stable bearing configuration, such as a tilting pad arrangement in a multistage centrifugal compressor, it may be susceptible to aerodynamic instability under high power levels when run appreciably above the first critical. Recently there have been a number of cases of aerodynamic excitation in centrifugal compressors causing severe stability problems.

Figure 25 represents the amplitude vs. speed of the NASA Brayton cycle experimental centrifugal compressors. The compressor is a single stage centrifugal unit with ball bearings mounted on flexible supports. When several incidents of failure were encountered in this system non-contacting probes were placed on the unit to monitor the rotor motion. Figure 25 shows that as the rotor speed approached the first critical speed around 17,500 RPM super-harmonic motion was obtained in addition to the synchronous motion. The superharmonic motion is approximately equal to the second critical speed. As the speed was further increased the amplitude at the second critical speed did not increase during acceleration. It was only upon a reduction of speed that the second critical was excited. There have been several incidents reported in which this phenomena is so severe that the amplitude upon reduction of speed increases at the second critical and does not reduce until below the first critical speed. As the speed was further increased to 52,000 RPM severe whirl instability was encountered at the compressor wheel as shown by the enclosed orbit. Figure 26 represents the NASA Brayton cycle test rotor at various power levels at 52,000 RPM. Figure 1 represents a superimposed picture of the rotor motion at no load and part load. Under no load the orbit is a small synchronous orbit. As the power level

is increased on the compressor wheel the orbit begins to increase as shown by Figures 1, 2, 3, and 4. For example in Figure 6 the amplitude would have caused a destruction of the rotor if the unit were not shut down. The whirl ratio observed in this orbit is approximately 1/5 of running speed, or around 10,000 RPM.

A number of incidents of self-excited aerodynamic instability have been reported in multi-stage centrifugal compressors. In a number of these cases it has been found that a change in bearing characteristics alone was relatively ineffective in stabilizing these rotor systems. However, one successful approach that has been used to stabilize rotors subject to aerodynamic instability has been the use of a squeeze film damper support system incorporated with the bearing. Figure 27 represents the rotor orbit of a seven stage turbo-compressor before and after stabilization with a squeeze film bearing. The upper left figure represents the unstable rotor orbit at a discharge pressure of 175 psig. If the discharge pressure were increased above this value the orbit would have caused destruction of the rotor. The upper right figure represents the stable rotor orbit at the full discharge pressure of 650 psig after introducing the squeeze film damper. Upon filtration of the orbit only a small component of fractional frequency whirl remained in the system. There have been several unsuccessful attempts to stabilize a turborotor suffering from aerodynamic instability by means of a squeeze film damper support. A recent analysis of the stability of a flexible rotor with aerodynamic cross coupling has shown that the support system must be carefully tuned to the rotor in order to promote stability. For example Figure 28 represents the stability characteristics for various values of support stiffness  $K_s$  vs. support damping. Values above the reference line 0 represent a real positive root and indicate an unstable system, while those below the line indi-

### MOTION OF A GAS BEARING ROTOR AT THE THRESHOLD OF STABILITY



CONDITIONS:

$$\omega_s = 5500 \text{ RAD SEC}, \Lambda = 3.5$$

$$\text{SCALE: 1 MAJOR DIVISION} = 100 \mu\text{in}$$

Figure 20. Gas Bearing Rotor at the Threshold of Stability ( $N = 52,500 \text{ RPM}$ ).



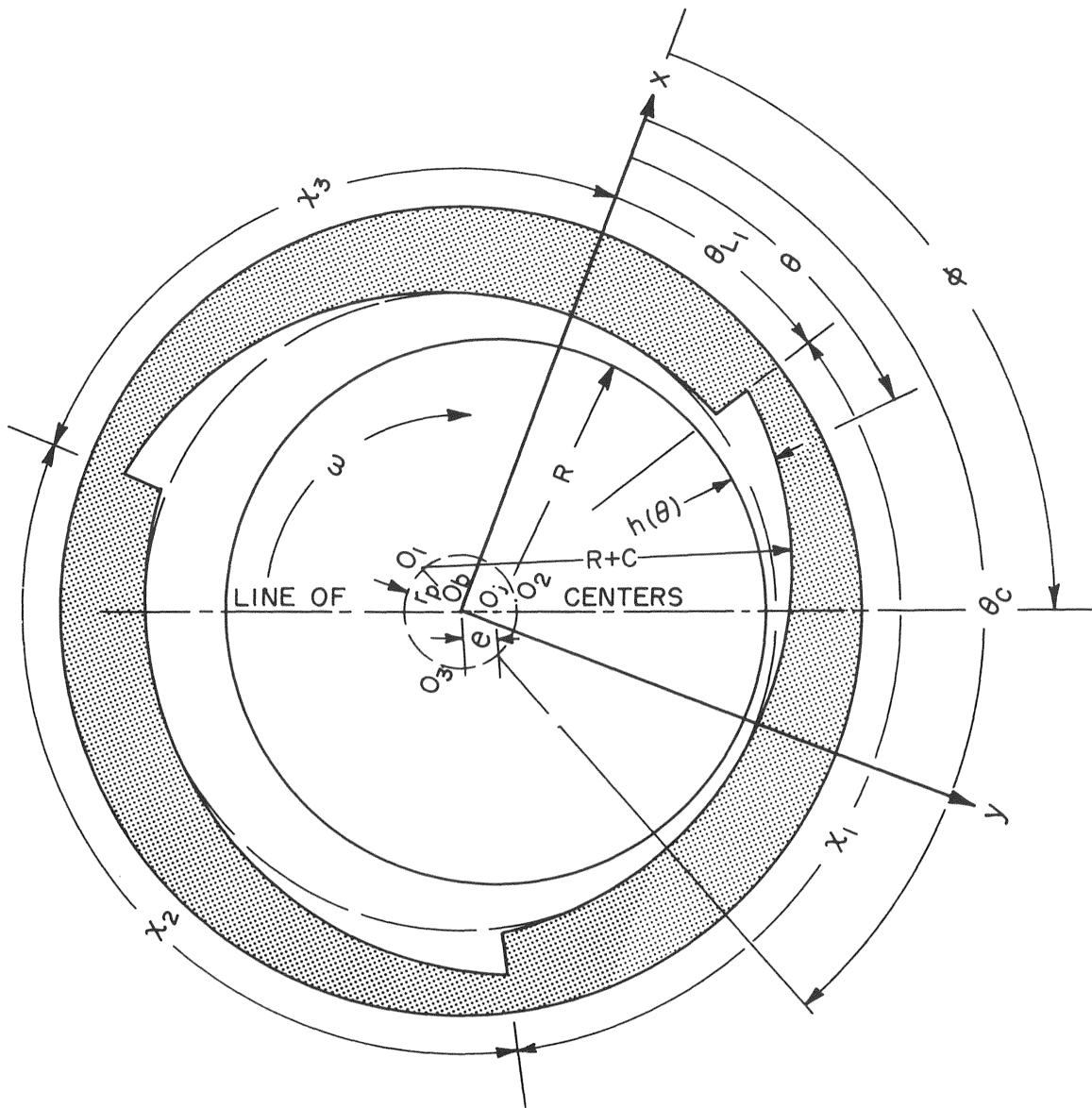


Figure 21. Multilobed Bearing Configuration.

cate stability. Note that the optimum stability is obtained in this case with a support damping of 1,000 lb-sec/in and a support stiffness between 50,000 and 100,000 lb/in. If the damping is either too small or too large the system will be unstable. Conversely if the support stiffness increases to 250,000 lb/in the system will also be unstable. In general, squeeze film dampers are highly nonlinear systems and if the damper clearance is not properly selected and the damper correctly aligned, the non-linearity can easily cause the damping characteristics to be excessive thus defeating the purpose of the support system.

#### SUMMARY

It is seen that there are many factors which can cause whirling in turbomachinery. Principal mechanisms of concern to the compressor design engineer are

hydrodynamic bearing instability, internal friction effect and aerodynamic cross coupling. These effects can cause serious self-excited whirl instability in a turbo-rotor which can lead to destruction of the system. If turbomachinery is to be built that is going to run several times greater than the first critical speed then it is important that the system be checked for stability and that proper electronic monitoring equipment be placed on the machine during operation so as to detect the possible occurrence of a dangerous whirl motion which may cause destruction of the machine.

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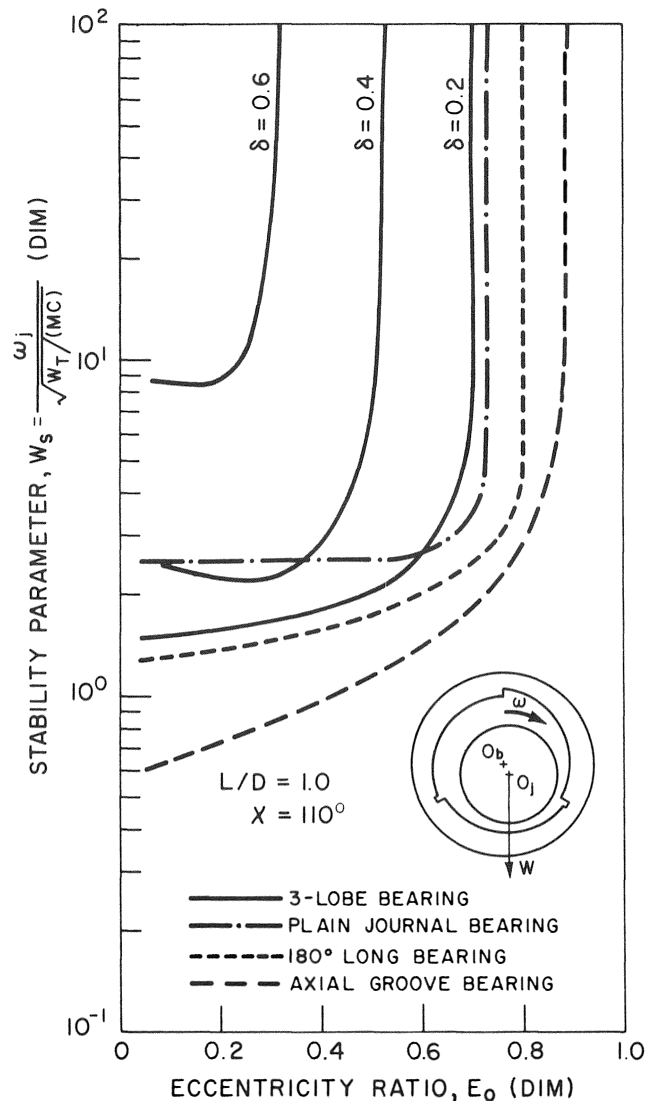


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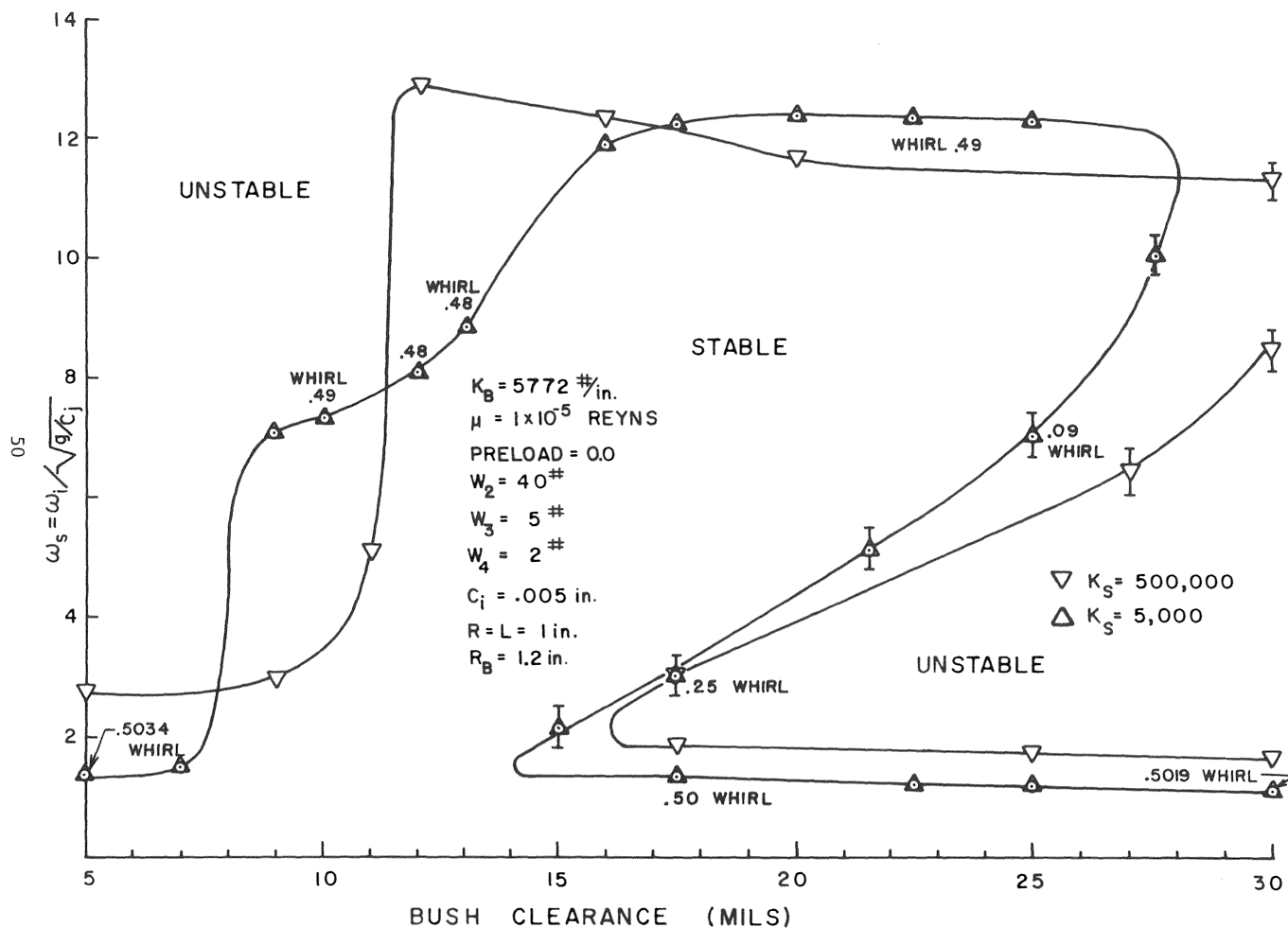


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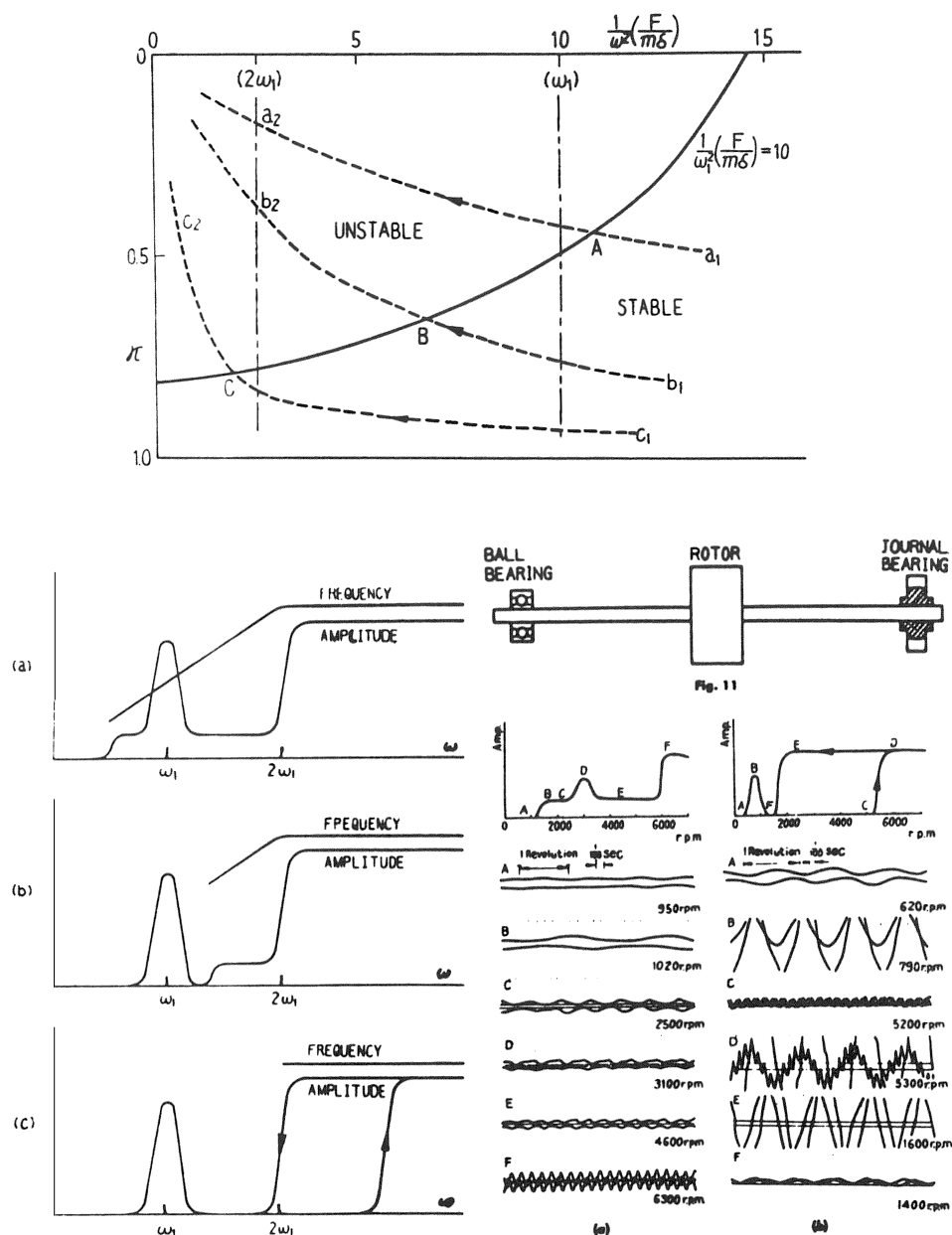


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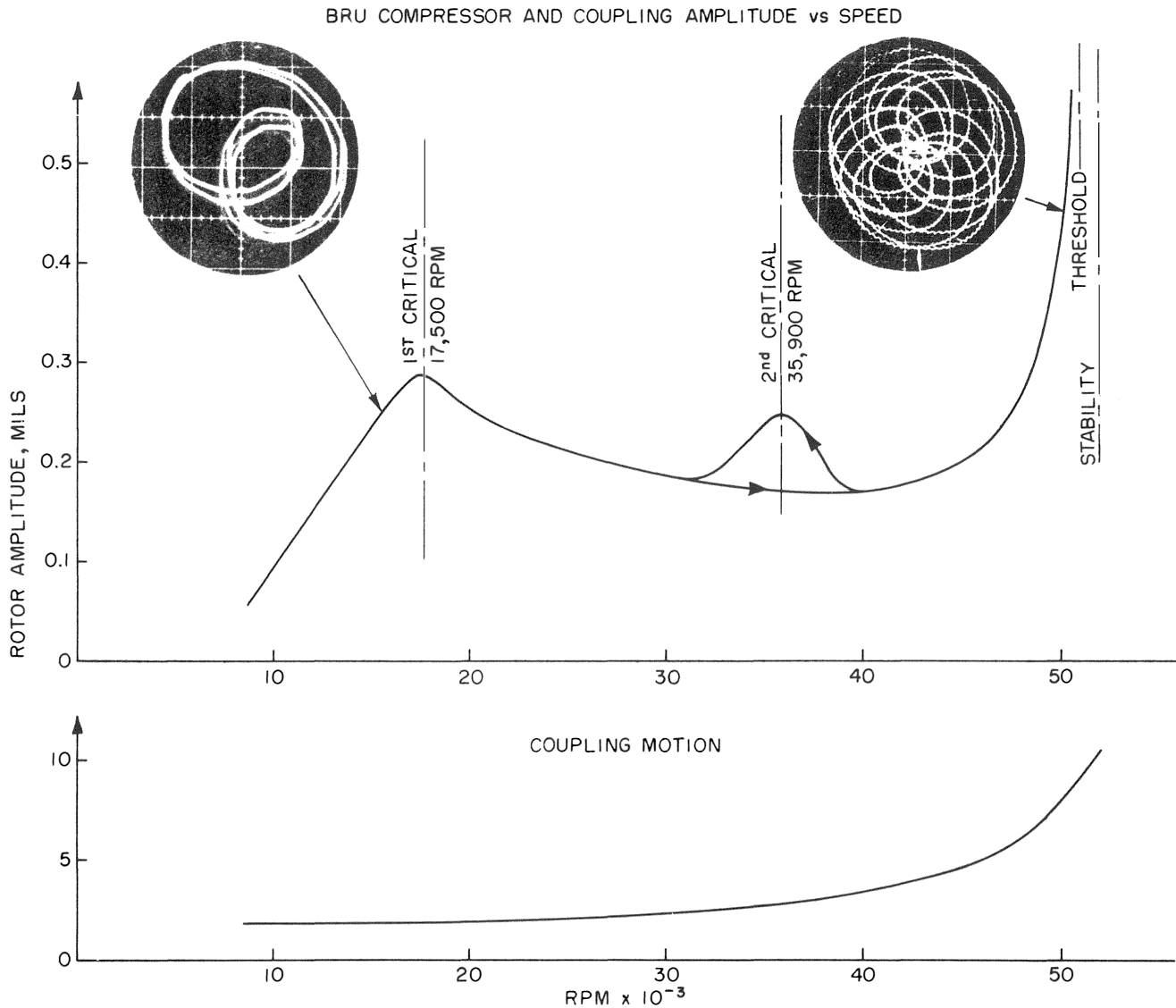


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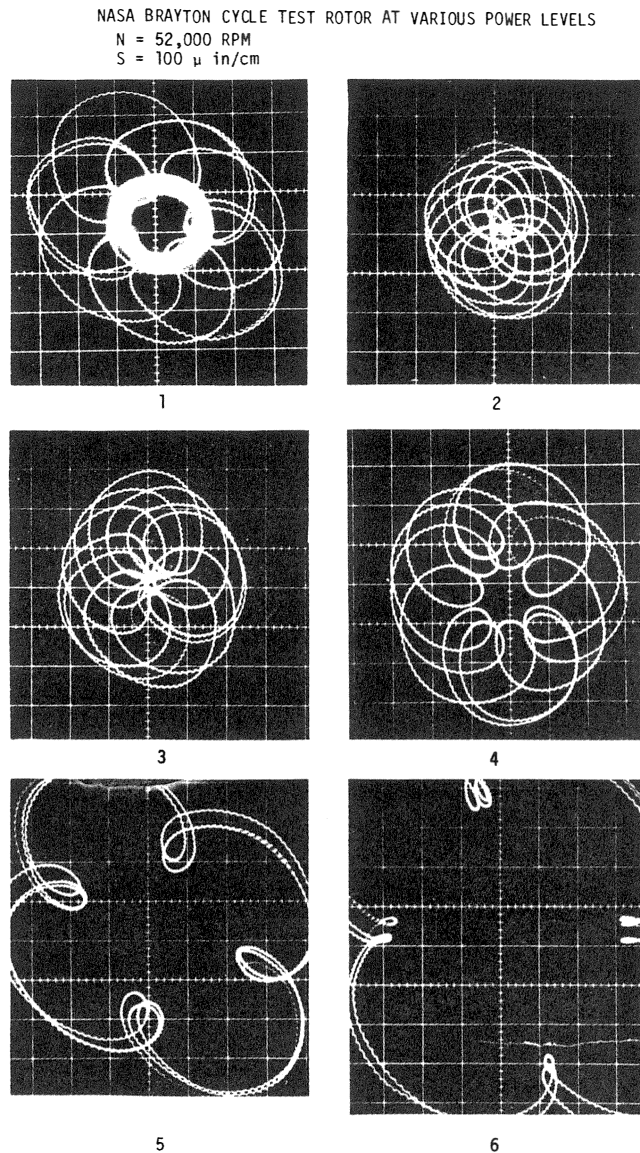


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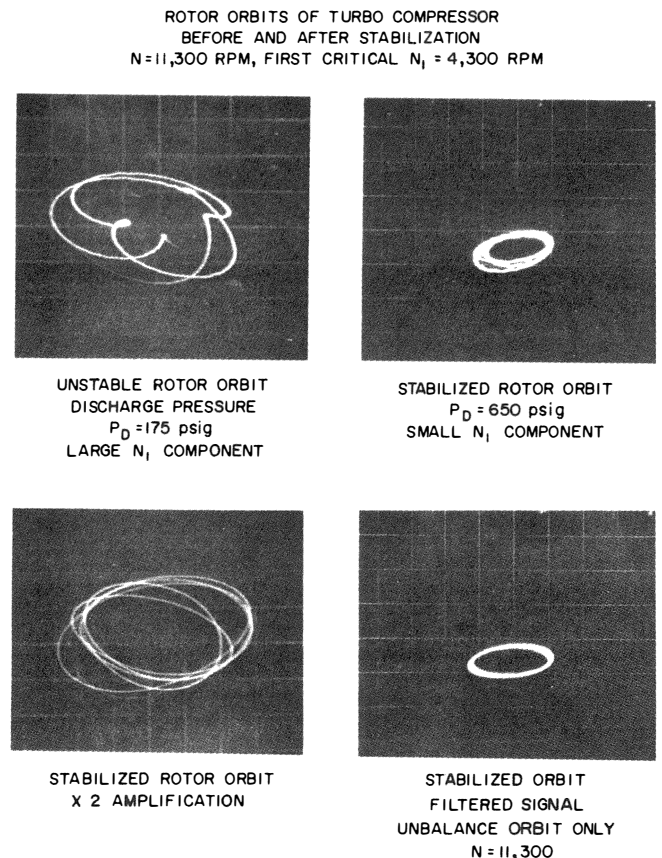


Figure 27. Rotor Orbits of a Turbo Compressor Before and After Stabilization.

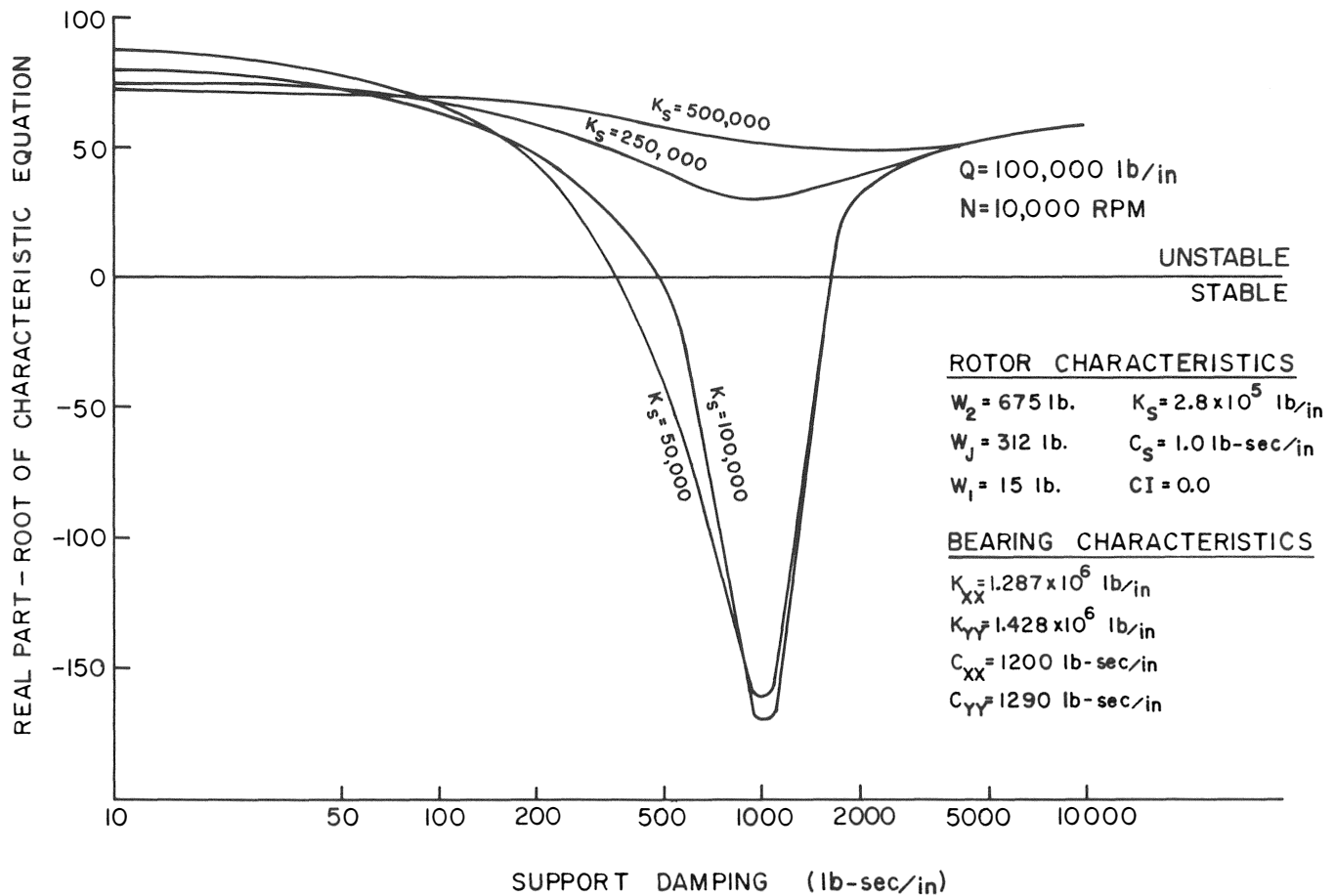


Figure 28. Stability of a Flexible Rotor With Aerodynamic Cross Coupling ( $Q = 100,000 \text{ lb/in.}$ ,  $N = 10,000 \text{ RPM}$ ).

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